Lesson 1 – Section 2.5 Angle Relationships

Creator: Heather McNeill
Grade: 10^{th} grade
Course: Geometry Honors
Length: 50 minutes

1. Prior Knowledge, Skills, and Dispositions: In this lesson, students should already have an understanding about what congruent, complementary, supplementary, vertical angles and linear pair of angles are since they covered these concepts in section 1.3. To understand the Linear Pair Conjecture, students need to recognize that any two angles which form a linear pair will sum to be 180º. From there students will be guided through discovery to learn about the Vertical Angles Conjecture.

2. Academic Content Standards:

Benchmark	Description
MA.912.G.1.3	Identify and use the relationships between special pairs of angles formed by parallel lines and transversals.
MA.912.G.8.1	Analyze the structure of Euclidean geometry as an axiomatic system. Distinguish between undefined terms, definitions, postulates, and theorems.
MA.912.G.8.3	Determine whether a solution is reasonable in the context of the original situation.
MA.912.G.8.4	Make conjectures with justifications about geometric ideas. Distinguish between information that supports a conjecture and the proof of a conjecture.

Students will be able to discover relationships between different pairs of angles.

Student will be able to practice measurement skills using a protractor.

Students will be able to develop inductive and deductive reasoning skills and practice cooperative behavior.

3. Description of Pedagogy: The lesson will be taught using a variety of techniques. Parts of the lesson will include independent work, partner, group and whole class work. These techniques will work fine for George; he has no problem working well with other male students, whom he already sits around and will be grouped up with.

4. Assessment: During the lesson, I will walk around the room monitoring students' progress, making sure each student is on task and participating in the task. During each section of the lesson I will ask the students to explain what they have found. From the student answers I should be able to judge where their level of understanding is and whether or not I should proceed onto the next section. I am looking to see that they understand that two linear pairs will sum to be 180º as well as how the Vertical Angle conjecture is formed and how it works. Once the lesson is complete I will assign the corresponding homework problems. (Section 2.5 # 1-10, 13-18) Their answers in reviewing their worksheet at the end of the period and the work they show in answering the homework problems should help me in understanding what the students left the lesson knowing.

5. Detailed Lesson Sequence:

What the teacher will do:	**Statements the teacher will say/Questions the teacher will ask:**	**Possible student responses:**
Warm-Up		**10 minutes**
Pass out the Warm-Up worksheet and put the matching transparency on the overhead. Title: Lesson Terminology Allow the students 5-7 minutes to fill in their 4 words. On overhead go over the 4 words. (3-5 minutes)	Please come in sit down and begin filling out the paper on your desk. You may use your book, and some of these terms may seem familiar to you. Be sure you also come up with an example or a picture that represents your description. This should help you in remembering what the words mean.	N/A
Engagement		**5 minutes**
Today we will be going over the linear pair conjecture and the vertical angles conjecture. Please take out a piece of paper and *without drawing any people*, draw your bathroom shower. You have one minute! You probably either have a stand-alone shower or a shower inside of a bath tub; draw what your shower looks like. How do you turn on and off the water? Where does the water come from the wall?	How do you turn on and off the water? Where does the water come from the wall? What does your shower look like? Did you draw the shower head? Be sure to include the shower head. (How else are you going to rinse the shampoo out of your hair?) Is your shower head attached to the side of the wall, or to the ceiling?	Yes/No. The side of the wall. The ceiling if a fancy bathroom, simulating rain.

Now take your protractor (some may need to be passed out for those students who didn't bring their own) and find the angle at which your shower head comes out of the wall or ceiling. Write the angle down in your picture. Now measure the other angle between the shower head and the wall.	At what angle does your shower head come out of the wall/ceiling? What else could we measure relating to the shower head? What is your second angle? Do your angles match your neighbor's? Why or why not? What do you notice? Why is that? Will this always be the case? Why or why not?	Multiple different answers. (e.g. 125º). The second angle made by the shower head. Multiple different answers. (55º). No, we drew our pictures differently. Even though our angles are different they sum to 180º. Yes this will always be the case considering the wall is straight and therefore forms a line. The shower head serves as the ray originating from the line (wall) and our two angles formed by our ray will sum to be 180º. We have created a linear pair of angles!
The Linear Pair Conjecture		**5 minutes**
Place the Linear Pair transparency on the over head (with the words Linear Pair covered up) and have a class discussion about how this picture is similar to the shower head engagement. Talk about other real-life connections.	What can you tell me about this picture? Does it remind you of something we just talked about? What is that? How is it similar? How is it different? What are other real-life examples using Linear Pairs, similar to our shower head example? Who will explain to the class	It has a line, a ray, two angles, letters, points, it is a linear pair of angles. Yes, the shower head example. We have a linear pair of angles, it is just oriented differently. A blade of grass coming out of the ground, a light switch on the wall, etc.

Formally state what the Linear Pair Conjecture is.	what they think a conjecture is? Who wants to try to state in words what they think the Linear Pair conjecture might be.	A generalization resulting from inductive reasoning. If two angles form a linear pair, then the measures of the angles will add up to 180º.
Vertical Angles Conjecture		**10 minutes**
Now we are going to move on to another type of angles. But before we do, I want you to put your protractors to the side, and draw two intersecting lines on this wax paper. (Pass out wax paper and sharpies.) (Place Vertical angles transparency on the overhead) Now label your angles as shown on the overhead, with the number one oriented at the top of your paper and number from 1 to 4 moving clockwise from angle to angle. Now I want you to label your four different angles, *without* using your protractors what do you think each angle measures? Write the degrees near the angle. Bring the class back together for a whole group discussion.	Now I'm sure we didn't all draw the exact same picture. Right? Look at your table partner's picture, is it exactly the same as yours? Great! Now *without* using your protractor I want you to label the angle measures for your picture. Think about what each angle measure is? Write each measure near its corresponding angle. Now I want you to trade wax paper with someone at your table. Look at your partner's picture and what they labeled for their angles, do you agree with them? Why or why not? Take about 3 minutes now to discuss your thoughts about each other's picture. If you wish, you may change what you wrote for the angle measures. Okay, now if you haven't already, please return your partner's paper. Everybody should have their own now.	Nope! Yes/No.

(The goal is that the students used what they know about linear pair of angles to figure out that each of their angles must be congruent to the one across from it.)	What did you and your partner talk about?	How certain angles must be the same. (Correct the word same with congruent, they are not the same angles, the angle measures are congruent.)
	Did anybody end up changing what they had down for their angle measures? Why?	Yes, I forgot that 2 angles must add up to equal 180º.
When it arises in the discussion that the vertical angles are congruent show the class how they can fold their wax paper over to see this. Fold the paper so that the vertical angles lie over each other.	Which angles are vertical angles? What do you notice about their measures? Is it weird that these angles are congruent? Why Not?	1 and 3 are vertical angles and so are 2 and 4. They are congruent. (Explain how using deductive reasoning we can see which angles must be congruent.)
Make real-life applications	What are some real-life examples of vertical angles?	Two roads crossing each other, the letter X, etc.
Formally state what the Vertical Angles Conjecture is.	Who would like to state what they think the Vertical Angles conjecture is?	If two angles are vertical angles, then they are congruent.
Ask which angles on the 2 transparencies are congruent.	Now looking at both transparencies, which angles are congruent?	Angles 1 and 3 Angles 2 and 4 Angles JKL and MKN Angles LKJ and NKM
Wrap-Up		**20 minutes**
Pass out student worksheet (Angle Relationships) and ask the class to begin working on it. They may work together as long as both people are engaged. If I see you simply copying answers we will all have to work individually.	I want you to first work on the side that says Angle relationships on the top, not the side that says Angle Relationships Practice.	

After about 15 minutes we will go over some of these. Wander around the room working with students and asking questions. Get the attention of the class and ask if they have any that they would like to see done on the overhead. Ask student to provide the reasoning and examples they came up with for numbers 3-6. Your homework for tonight is Section 2.5 # 1-10, 13-18	How did you figure that out? Did you use a conjecture? Which one? Why did you choose that one?	

6. **Materials:** Pencil, blank paper, wax paper, protractor, Discovering Geometry textbook, blank transparencies, overhead markers, class worksheet, Conjecture transparencies, sharpies

7. **Resources*:**
   http://www.glencoe.com/sec/math/prealg/prealg05/study_guide/pdfs/prealg_pssg_G081.pdf

   http://education.ti.com/educationportal/activityexchange/Activity.do?cid=US&aId=8670

   http://www.pas.k12.mn.us/1936208261569450/lib/1936208261569450/parallel_lines_and_vertical_angles.pdf

8. **Lesson Starter:** See attached PDF. It is the pages from the teacher addition in their textbook.

Analysis of Teaching Event 1

For my first lesson in the high school math classroom, I taught a Geometry Honors class with primarily tenth grade students. The concept of the lesson was Angle Relationships and the goal was for the students to learn about the Linear Pair Conjecture and the Vertical Angles Conjecture. The three main learning objectives I had for this lesson were that the students would be able to discover relationships between different pairs of angles, practice measurement skills using a protractor, and develop inductive and deductive reasoning skills as well as practice cooperative behavior.

For my student artifacts I was able to collect ten different student homework assignments and the recording of my lesson. After looking through the different questions and answers, I believe that the first five questions were an easy review for most of the students. They have seen similar diagrams before and they were able to apply their new conjectures to find the measures of the angles. I feel that question number six was a key question to show the level of understanding of the lesson on linear pair of angles. It asked, what's wrong with the picture where points A, B, and C are collinear but the two angles only add to be 170º. This question requires students to know that a linear pair of angles must sum to be 180º and although looking at the picture it appears that the two angles combine to equal 180º the students must not assume, and instead add the two angle measures together. Seven of the ten students wrote down an acceptable answer expressing that the two angles should sum to be 180º, but the two angles in the picture did not (see Jacob, Kayla or Darby's homework). I also feel that number nine from the homework assignment is an important question in showing the students' level of understanding of linear pairs and what a converse and counterexample are. The question is, you discovered that if a pair of angles is a linear pair, then the angles are supplementary. Does that mean that all supplementary angles form a linear pair of angles? Is the converse true? If not, sketch a counterexample. This multi-question format seemed to be too much for the students because looking at their answers it seems that they answered one part or the other of the question. Some students were correct in the parts of the question they did answer though; many students drew a counterexample showing two nonadjacent angles that sum to 180º. This way the angles do not form a linear pair, but are supplementary. This type of solution can be seen in Jacob's, Kayla's, Kennie's, Sienna's, and Darby's papers. However, Natasha wrote that the converse is true, and she did not draw a picture. If she would have drawn out pictures maybe she would have seen what the question was asking. Kodiak left a question mark for question nine. He possibly didn't understand what the question was asking or didn't know how to begin answering the question. If I were the classroom teacher I would go over this question as a class the following day because I feel that this is a question that covers many aspects of the lesson.

Since the class has seen different examples of vertical angles and supplementary angles before, this lesson was somewhat of a review for them, however, the new aspects were the formal conjectures. Being able to state what the Linear Pair Conjecture is and to understand its similarities and differences to supplementary angles, as well as formally stating the Vertical Angles Conjecture. I feel that these subtle, but important new details were understood among the class. I was excited when we would have a whole class discussion and the students would participate, such as twenty-two minutes into the recording, when the students provided different real-world examples of linear pair of angles. This told me that they knew what was going on enough to think of everyday objects that represented the same idea. Just after twenty-eight minutes into the lesson you can hear some of the students' discussions when asked to talk with their partner about what they believed to be the angles on their wax paper. As I walked around I could hear comments such as, I agree with you because you labeled the larger, obtuse angle a higher number than the smaller angle. Jacob actually compared his guessed angle measures to what his protractor showed and saw that the two angle measures didn't add up to 180º. On the video you can hear him questioning why this is, because he knew that they should sum to 180º. This made me happy to hear that he was confident enough to question what the protractor was telling him (again at twenty-eight minutes in).

In completing my analysis of the level of students' understanding from the lesson, I modeled many of Carpenter and Lehrer's (1999) points in developing understanding. I believe that I had the students articulating what they knew in the warm-up with writing down their own descriptions and pictures related to the vocabulary terms. In having the students write a description and then draw a picture or give an example, it provided them with multiple representations for the same term, providing further connections. I feel that the lesson extended and applied mathematical knowledge when we discussed as a class the similarities and differences between supplementary angles and linear pairs, as well as deriving how the vertical angles conjecture is true by using what we knew about linear pairs. Finally, the students were able to make the mathematical knowledge their own when they developed real-world examples of the mathematics. I believe that making these connections is the most important part. Bridging the gap between what is learned in school and what happens in the real-world is crucial and the closer we can bring those two systems together the more meaningful the mathematics can be. It's one thing for the teacher to list a whole bunch of real-world examples, but I feel that when students come up with their own connections the content can then be understood. Some students' examples may be a little strange, as in a student's example of a chair as an example of a linear pair of angles (at twenty-two minutes into the recording.) In forming a list of examples to share with the class I would have never thought of a chair leg coming off of the base of the chair, but for that student, the idea works and that's what will help them remember linear pairs of angles. I also feel that the students were able to reflect on what they knew when they had to decide on their angle measure on their wax paper, then switch with a partner and discuss whether or not they agreed with the chosen angle measures. This required the students to use what they knew to decide the reasonableness of the angle measures.

Looking back at the lesson as a whole, I feel that it was a valid first attempt; however, there are many aspects that could be improved upon. In revising this lesson I would build in more time to answer questions and for class discussion about what is going on with the different activities. I felt that the lesson was a mile wide but only an inch deep. Although it only covered two topics, I wanted to present linear pair of angles and vertical angles in multiple ways, which I feel I did by providing multiple real-world examples and applications to everyday objects, however, under the pressure of time I felt that I was not able to give enough time to respond to each student's question. Many times I would ask for an answer and call on a student who would share their answer aloud but I would never validate whether their answer was correct, incorrect, or partially correct and how it could be better. Instead, since it wasn't the exact answer I had on my key I called on another student hoping to hear the 'perfect' answer (See six minutes in). In watching the recording of my lesson I noticed that I would ask a question, get an answer that wasn't exactly right and would simply move on telling the class what I, the teacher, had. This was not good. This would instill in students that the only 'correct' or 'acceptable' answer was the answer the teacher has. Another revision I would make is in classroom management, I feel that if it were my 'real' class and I saw these students on a daily basis that I could lay down my set of expectations for classroom behavior. I didn't appreciate students getting out of their seat without permission. In the future, I will not attempt to talk over the class. I will wait until it is quiet and I have their attention before speaking. I also noticed that I tried giving directions while passing out materials (See twenty-three minutes in). In doing so my sentences were broken into parts because I would become distracted. There were no students listening to my directions and if there were they probably would have been confused because they were not clear and direct. Then, once I was done passing out materials, I had to announce the directions again. If I would have simply passed out the materials without trying to multitask things would have been better. My questioning has room for improvement as well, I feel that I asked a variety of different types of questions, from lower to higher order questioning, but forgetting my wait time and the way in which I phrased the questions was crucial. Numerous times I found myself asking, “Does anybody want to describe…” as well as “Does that make sense?” in reality these are yes/no questions and won’t tell me anything about what the student knows. I also should try to avoid asking a question and leading to the answer, or even answering it myself. In the recording I said, “Both are valid, right? Of course the class will simply agree with the teacher, anything to move on and be done with the lesson. If I were to teach this lesson again I would have an example of a shower that I would want them to draw. Initially I didn’t want to provide them with a picture because I wanted each person to be creative and not copy what was on the overhead; however, there was some confusion as to what to draw and what should have only taken a minute required five minutes. Thus in the future I would put an example up so that they could have something to reference. In the vertical angles part of the lesson, when I asked the class to switch their wax paper with a partner and then to discuss whether they agree or disagree with the angle measure their partner put, the students simple told each other “agree” or “disagree”. In revising this lesson I would be sure that the student have a dialogue and talk about the reasons why they agree or disagree with their partner's decisions. I feel that the talking in the classroom should be mainly students, on-task talking about the mathematics, not necessarily the teacher's voice. With this said, there were times when I would make a comment, for example, "So you know all the way around is 360º." When I could have instead asked the class what they knew about what they saw. In having the students drive the conversation you are not restricting what is said and therefore are more likely to see and hear what the students know. The last thing I would want to correct in teaching this lesson is that I would have more time to go over the student worksheet. I ran out of time in the end and had to quickly go through the answers for the four questions. I feel that if the before mentioned changes are made then more time would be available in the end to discuss student answers for the worksheet.

I thoroughly enjoyed teaching the lesson and feel that students did learn about angle relationships and gained an understanding of the material. I am glad to have had the opportunity to teach the lesson and learn for myself what is required to implement a successful lesson.