[Algebra I ~ Linear Relationships]

Standards- and Research-Based Study of a Curricular Topic

Section and Outcome	Selected Sources and Readings for Study and Reflection Current Research on Topic
I. Identify Adult Content Knowledge	Linear relationships are a part of Algebra which is just one piece of the giant framework of mathematics. The way in which we think about and apply mathematics helps us to represent data, manipulate relationships, and solve problems; All of which are skills necessary for working with linear relationships. Observing change and working with variables are two important topics necessary for linear relationships that we will uncover. As I read the research I noticed that I began to think about mathematics differently. All through school I found enjoyment when my teacher would present us with a crazy math problem requiring us to find Johnny's age when he is 3 years older than his sister who is half the age of their cousin who is 14. Now I see that I am the weird one. Most students hate these types of problems and shut down when a teacher presents them with anything similar. The reading helped me to understand that sometimes students just need the meat of the problem. They need the overarching concept made clear. Let them solve the equation to find Johnny's age instead of presenting a maze of clues about every member in Johnny's family. If a teacher does want to bring real-world material into the problem then the teacher should make it meaningful to the students. Don't figure out Johnny's age, rather figure out the number of pizzas to order for the pizza party they are having on Friday. An additional real-world example that students might be familiar with include: deciding which account they should put their money in to get the greatest return. An alternative to getting students attention through real-world math problems could include discussing the historical perspective of the mathematics. This was something new I learned from the readings. I thought it was interesting that though mathematicians were using algebra, they went 750 years without the use of variables. Students with an interest in history could then find a link to the math. The enduring understandings that all adults, including teachers should know about this topic are defined in a three phase format. The first is representing some aspects of things abstractly, the second is manipulating the abstractions by rules of logic to find new relationships and the third is seeing whether the new relationships help us understand or relate back to our original thing. During the process of abstraction one should look for patterns between two different things. Then one should create a symbolic representation to represent this abstraction that can be used to help make sense of the observed pattern or relationship. With a symbolic representation in hand one can then try to manipulate it using the rules of algebra in an attempt to reach an end goal. We use symbolic relationships to help us to better see the relationship between two things. In comparing the two quantities we can use a table, equation, or

	graph, just to name a few tools. Many times what people find more interesting than the relationship between the two quantities is the rate of change of one quantity which may or may not directly depend on the other quantity. This ties back into observing the relationship and manipulating it to find new information. The readings help me to see what a K12 education is aiming for by breaking down the big picture, mathematics, into its main ideas. To 'do' mathematics one must follow the three phases as described above. By taking part in the three phases of mathematics students are using their skills to discover relationships, find ways to represent these relationships, learning to work with and manipulate these relationships so that they can then interpret information. Thus a K12 education is aiming to teach students the skills they need to be able to accomplish these three phases of 'doing' mathematics. When students can successfully accomplish the three phases they are able to apply these skills to other situations outside of the mathematics classroom. The reading in *Beyond Numeracy* does an exceptional job at clearly explaining that the basic understanding of algebra is knowing that things must be in harmony, balanced. If someone changes something they must do so everywhere as to restore the balance. Anytime a teacher explains a concept in a manner that brings the real-world into the classroom they are automatically making the lesson more interesting. Students must first decide that they want or need to learn what the teacher is teaching and when the teacher shows the students that what is being taught is applicable to them, their need to know that this is important to them is met which allows them to begin listening. Paulos suggests that when the mathematics teacher introduces variables, an entity of math much disliked, they should compare them to pronouns in the English language. Nouns are like constants in math, while variables are like pronouns. Just as pronouns make English much more versatile and many times simplified, variables play a similar role in mathematics. Algebra is used in many areas, including business, science, engineering, and day to day living. Paulos explains a real-world example about having to choose what type of account to keep his money in. The two different options varied in the amount they charged and the amount that they returned. Algebra was necessary in determining the better option.
II. Consider Instructional Implications	In *Benchmarks for Science Literacy*, the authors explain that there is no agreed upon answer to how much algebra one should know. One year of algebra has become the norm after trends showing a correlation between taking algebra and the amount of vocational success one has in life. Suggestions that were provided in the readings to aid in effective instruction include a deep understanding on what some call basic concepts. Students must first understand what symbols are; tools that are used in many forms and situations; both in mathematics and outside of mathematics. Once students understand what symbols are, they then need to know how to manipulate them.

	Symbolic representations are useless unless one has the knowledge and skills to manipulate them properly. Difficulties and misconceptions are definitely tied to this topic and it is important that teachers are aware of them. It is important that students understand what their solutions mean when working with algebra. Many times the sets of numbers, letters and operations seem to cloud the student's understanding and this unfortunately affects their understanding of what their solution means. It is important not to present too much to the students in their learning phase. Initially students should focus on the relationship between one variable and another. Teachers should first teach students how to create and interpret data in a table and then how it behaves when graphed. Once students are experienced in this, they can begin to work backwards without being overwhelmed. It is important that students understand that graphs of data can help them to more easily identify patterns and relationships in the data. Benchmarks for Science Literacy suggests that a practical way to teach students about the relationship between equations, table and graphs is through the use of a computer spreadsheet and graphics software. In doing so, encourage students to use data that is interesting and meaningful to them on a personal level. In discussing the findings when working with relationships in science it is important to have a discussion on reasons why the data may not fit perfectly; whether it is due to human error, choosing the wrong formula to represent the data, or inappropriate ranges. When each of these reasons are accounted for there is the almighty reason that the world will not always fit the mathematics perfectly. By grade 2 students should be able to understand that sometimes a change in one thing causes a change in another thing. By grade 5 students are able to describe how changes in one variable affect changes in another variable. Through the use of graphs and tables students can explore relationships between two different variables. From grade 6 through grade 8, students are expected to come across variables, know what they are, what they mean and what their purpose is. It is important that when teaching variables it should be done through real-world situations, not abstract. Involve situations familiar to students so that they are more likely to understand and be interested. By the time students reach high school they should have the foundation and skills to become proficient in translating between a table, a graph and an equation. Students will encounter different functions with varying relationships. With these skills students are able to project and explain data from inferring relationships they see either from an equation, table or graph. *Benchmarks for Science Literacy* discusses how students will encounter that algebra is used in design, as well as the natural and social sciences. Teachers should make clear that symbolic equations can be represented through words, graphs or tables and real-world applications can be made in the fields of physics, finance, engineering and many other applicable topics. The K12 big picture view of algebra that students need to be comfortable with are the understanding symbols, how to manipulate them, and how to make sense of what the symbolic representations are telling you. Eventually students should come to understand that these symbolic representations can also be viewed as tables, graphs or words. It is also important to teach students the concept of change. In the elementary grades students should notice change and begin to

	describe it. In middle school students should begin to understand linearity and how slope represents the rate of change, this way by the time students reach high school they are prepared to learn about other types of functions, ones that are not linear.
III. Identify Concepts and Specific Ideas	*Benchmarks for Science Literacy* explicitly states that while students are not expected to memorize formulas, they are expected to understand proportionality, how to read an algebraic formula, and how to interpret a graph. Learning goals that align well with this topic include exploring the idea of change is important and crucially related to linear relationships. Students must understand how two quantities relate in order to understand how one is affecting the other and thus causing a change. Additionally, working with variables within ones symbolic representation is important. Many times when working with linear relationships we have unknowns and we must know how to properly manipulate our equation in order to find our missing information. Balanced, balanced, balanced equations. Understanding that equations must stay balanced is a crucially important concept that many students seem to struggle with, as well as the skill of knowing how to interpret a graph and how to graph data. With these goals defined we are able to determine that spending time working with creating symbolic representations is important, and that other things like identifying the type of function is less important. If students are able to represent a function and view it's graph they can then get a good idea of the type of function. Explicitly teaching function recognition can come later. The learning goals in the *Benchmarks* compare to the ideas in *Principles* in the following ways. Patterns may show up in many places, including nature. Many times we will see that changing one thing will cause a change in another thing. We can use tables and graphs to help display these relationships. Mathematical representations in the form of an equation can be used to represent these relationships. Rates of change can be calculated from differences in the magnitude. The rate of change can vary. Different mathematical models may be more appropriate depending on the situation. We can use tables, graphs, equations, etc. Viewing graphs can help us to more easily recognize patterns or varying rates of change. Some features may stay constant while others are changing. We can have constant rates of change or irregular rates of change, or a combination of both. We can use symbolic equations to summarize how quantities relate to one another. Embedded in the standards across all grade levels is the idea of students comparing more than one quantity to find a relationship. They are to then represent that relationship in some form of a mathematical model. The sophistication of the model and the depth of understanding will directly correlate to the student's grade. Below are the overarching standards for algebra which are the same in all grade levels: - Understand patterns relations and functions - Represent and analyze mathematical situations and structures using algebraic symbols

	- Use mathematical models to represent and understand quantitative relationships - Analyze change in various contexts Since the overarching standards for each grade level are the same for algebra the expectations that are listed beside each standard help to clarify exactly how the standards are being addressed. For example, in PreK-2 analyzing change in various contexts may mean describing that a person is growing taller. Thus over time the height increases. While in grade 6-8 the same standard is addressed through the use of graphs to analyze the nature of changes in the quantities in a linear pair. As the grade level increases the depth of knowledge for the standard also increases.
IV. Examine Research on Student Learning	Students often struggle with the use of variables. Many times, even in the secondary grade level, they don't understand the role variables play and stumble on the seemingly random assignment of variables. Additionally, students struggle with graphing. They often have a tough time scaling their axes appropriately and understanding how the scale they choose will change the way their graph looks. Students have a hard time understanding the relationship between the two quantities graphed and the relationship that is displayed through the data. When the data is graphed they struggle to understand the big picture that is displayed. Many students rather focus on one point at a time. Recognizing change over time can be difficult for students. There are a multitude of possibilities that can cause a student to have misconceptions. Students may believe that variables stand for a single specific value, or as generalized numbers before they really understand the true meaning and purpose of variables. It has been shown that students with long term exposure, three or more years, to working with computer software increases a student's understand and ability to work with variables. It is suggested that having students work with Microcomputer-based laboratories (MBLs) can help students to better interpret graphs. Student can improve their understanding of the relationship between the two graphed quantities and know what the data represents, in the case of distance and time the graphed data shows us the rate of change, in this case the speed. Often students know how to manipulate equations to get what they want, but do so without a proper idea of equality and what the equal sign really means. Why is it important to do unto one side what you do to the other? It is suggested that elementary students work with the computer software to begin their development of the idea of variables while MBLs be used with middle school students to understand relationships between graphs. The idea of equality should be addressed in the primary grades and can be taught through multiple mediums, not just values printed on a piece of paper. In the elementary grades students tend to only observe changes in one quantity at a time. Therefore comparing two different quantities can be a lot to ask. In fourth grade students can begin to compare two quantities, but the data must be in numerical form listed in a table, as opposed to a graph. The research draws attention to important pre requisite knowledge by explaining the skills students should come to class knowing. For example, when the teacher

	talks about intercepts on the graph, it should not be the first time that students have ever seen a graph. Students must know how to work with variables as well as read and interpret graphs in order to succeed in working with linear relationships. The concept of equality is an elementary skill that must be understood in order to work with linear equations. By the time students are learning about linear relationships they should be able to properly compare two quantities and interpret the relationship between the two. The research can be used to compliment the benchmark ideas in a number of ways. Teachers can expose students to the idea of variables in different fashions, including but not limited to the computer. The research stresses that it is important that teachers ensure their students sound understanding about graphs, how they are created, read, what they mean, etc. When working with equations that represent our linear functions we must check that when our values are plugged in that the equation holds. The value we get for the left side must equal to value we get for the right side. If the equation is not equal then we know the values we used are not on our line. Using the research, teachers should scaffold their teaching to support the idea that students may not be as strong at taking information from a graph as they might a table. The teacher should work to help students on this skill to improve on it. It can be beneficial to question students about their prior knowledge about a topic prior to teaching it. It can help to ask students what they understand equations to be. Again, the idea of the equal sign representing equality from one side to another is challenged by students' lack of understanding. The research shares that to help students avoid misconceptions teachers can allow students to see that they can solve problems in a variety of ways and they can then manipulate the equations they get for answers to get the same thing. Their two equations, though set up to represent different things, are essentially the same. It is suggested that students work with isolating variables in equations, this causes students to feel differently about the equation and they can then better focus on what stays the same and what is changing.
V. Examine Coherency and Articulation	The map helps a teacher to trace a concept or skill from its simple beginning in K through 2 to a culminating, interconnected , sophisticated idea attained by the end of grade 12 because the teacher can see what the student has learned at each grade level. While students are learning the same standards from K-12, - Understand patterns relations and functions - Represent and analyze mathematical situations and structures using algebraic symbols - Use mathematical models to represent and understand quantitative relationships - Analyze change in various contexts the depth in which the standards are addressed varies. With the map teachers can see this depth across the grades. Initially students are identifying that a change is happening, while later they are calculating that change and describing what it means. It is important for teachers to see the building blocks on which students' concepts are founded on. This helps teachers who teach at the secondary level to understand the process in which students got to where they

	are today. Students must be able to understand change when comparing two variables, they must be able to represent this in multiple ways and understand how an equation, graph and table are three different ways to display the same information. Being able to create and work with symbolic representations and understand change are two important skills applicable across many areas outside of mathematics. Science, engineering, and business all use these skills in collecting and interrupting data during research, marketing and development. In viewing the conceptual strands on the map teachers can see when students learned the necessary prior knowledge that they must apply to succeed at a current task. With this knowledge, teachers can properly review or scaffold their lessons to support this level of understanding. The strands also help teachers to see how the standards relate to one another; thus allowing teachers to create meaningful lessons that includes multiple standards. The map and its narrative section help me to better understand the topic and how it is taught over the course of thirteen years. While students may be introduced to 'slope' for the first time in Algebra I, the reality is that they have talked about it prior to Algebra I, just in other terms that were less technical.
VI. Clarify State Standards and District Curriculum	According to the state standards students are expected to know how to write equations, create graphs, and solve equations for specific pieces of information. An important benchmark in the standards that was addressed in the earlier sections of the CTS is having students view a graph and extract information from it, including the equation. In the 9-12 standards I don't see where the standards explicitly address working with variables and what variables mean. This is addressed in the grades 6-8 section, but we know from the CTS that it is important that even high school teachers go over variables. The verbs used for this topic include to graph, solve, write, and determine. I believe that these verbs are all appropriate for what students are asked to know how to do. Having students determine pieces of information about a function by looking at the graph is supported by the research and would present an appropriately high cognitive challenge to students. Before working through the CTS I had never thought about the big ideas related to working with linear relationships. I knew that in working with linear relationships we use equations, graphs and tables, but I didn't consider the overarching concepts of equality and change. Now I understand these overarching ideas and can plan to address them in my teaching. Learning goals integral to this topic include knowing how to work with equations, write them, manipulate them and solve them. Students must also understand how to graph, what a graph shows and how it is constructed. Lastly, to be successful with this topic, students must understand the relationship between a liner function as it is represented as an equation, table or graph. Reading the earlier parts of the CTS helped me to better understand the intent of

	the standards because I was able to understand the foundation in which the standards are based. I now know what students learn earlier in school and what they are expected to know when they reach high school. The three enduring ideas from section I were that adults know how to represent information, manipulate information and draw logical conclusions from the data. All three skills are definitely taught through the standards of linear relationships. Students are to create equations, manipulate them to get specific pieces of information, and then use that new information to help understand a different aspect of the data. Knowing what I know now, from sections I-V, I am better able to help my students understand the state standards. If they struggle with a particular concept I am more likely to know why they have the misconception they have and how to correct it. Prior to learning linear equations, the district has students learning slope and rate of change, this is great because we know how crucial change is when discussing linear relationships. However, they do not discuss variables and how they are to be used when working with the linear equations. I would include lessons on variables, what they are and why we use them. This way we can relate this knowledge to our use of variables like x, y, m, and b when discussing linear equations. I believe that each topic is important, it exists because it was a necessary tool at some point down the road and it should not be skipped. Additionally, students learn differently and what may not be essential to understanding for one student may be very instrumental to another student making sense of a concept. Before diving into teaching linear relationships, it is important to check your students' understanding of the three big ideas, if they are not comfortable with the big ideas then the instruction should change gears momentarily while students are brought up to speed and gain appropriate levels of understanding of the big ideas before the teacher proceeds with the cookie cutter pacing guide. Again, I believe that while the concept of change is addressed across all grade levels, variables are not. I think that once students reach high school we assume that they understand the concept of variables, when many still struggle. It is important that this topic be revisited in order to help students best understand linear relationships and algebra as a whole.