## Lesson 2 - Section 4.1 Triangle Sum Conjecture \& 4.2 Properties of Isosceles Triangles

## Creator: Heather McNeill

Grade: $10^{\text {th }}$ grade
Course: Geometry Honors
Length: 50 minutes

1. Prior Knowledge, Skills, and Dispositions: In this lesson, students should already have an understanding about triangle properties since they learned about triangles in section 1.5 , however in this lesson we will formally cover the Triangle Sum Conjecture, the sum of the measures of the angles in every triangle is $180^{\circ}$. From there students will be guided through discovery to learn about the properties of isosceles triangles and the Isosceles triangle Conjecture. To be successful in this lesson, students must know how to measure angles using a protractor.

## 2. Academic Content Standards:

| Benchmark | Description |
| :--- | :--- |
| MA.912.G.4 | Identify and describe various kinds of triangles (right, acute, scalene, isosceles, etc.). <br> Define and construct altitudes, medians, and bisectors, and triangles congruent to <br> given triangles. Prove that triangles are congruent or similar and use properties of <br> these triangles to solve problems involving lengths and areas. Relate geometry to algebra <br> by using coordinate geometry to determine regularity, congruence, and similarity. <br> Understand and apply the inequality theorems of triangles. |
| MA.912.G.4.1 | Classify, construct, and describe triangles that are right, acute, obtuse, scalene, isosceles, <br> equilateral, and equiangular. |
| MA.912.G.8 | In a general sense, mathematics is problem solving. In all mathematics, use problem- <br> solving skills, choose how to approach a problem, explain the reasoning, and check the <br> results. At this level, apply these skills to making conjectures, <br> using axioms and theorems, constructing logical arguments, and writing <br> geometric proofs. Learn about inductive and deductive reasoning and how to <br> use counterexamples to show that a general statement is false. |
| MA.912.G.8.1 | Analyze the structure of Euclidean geometry as an axiomatic system. Distinguish <br> between undefined terms, definitions, postulates, and theorems. |
| MA.912.G.8.3 | Determine whether a solution is reasonable in the context of the original situation. |

- Students will be able to explain that the sum of the measures of the angles of a triangle is always $180^{\circ}$.
- Students will be able to describe a relationship related to the base angles of an isosceles triangle and make a conjecture about triangles that have two congruent angles.
- Students will be able to show problem solving skills and inductive reasoning skills as well as the implementation of new vocabulary.

3. Description of Pedagogy: The lesson will be taught using a variety of techniques. Parts of the lesson will include independent work, small group and whole class discussions. These techniques will work fine for George; he has no problem working well with other male students, whom he already sits around and will be grouped up with.
4. Assessment: During the lesson, I will walk around the room monitoring students' progress, making sure each student is on task and participating in the task. During each section of the lesson I will ask the students to explain what they have found. From the student answers I should be able to judge where their level of understanding is and whether or not I should proceed onto the next section. I am looking to see that they understand that no matter what type of triangle they have the sum of the angle measures will always be $180^{\circ}$ as well as properties of isosceles triangles; specifically, characteristics about congruent sides and congruent angles. Once the lesson is complete I will assign the corresponding homework problems. (Section 4.1 \# 2-12 even, 16, 20-25 and 4.2 \#2-10 even, $14-24$ even) The questions they ask in class and the answers they give me through class discussion, as well as answers in reviewing their homework which will be returned the following class period. The work they show in answering the homework problems will help me in understanding what the students left the lesson knowing.

## 5. Detailed Lesson Sequence:

| What the teacher will do: | Statements/Questions the <br> teacher will say/ask: | Possible student responses: |
| :--- | :--- | :--- |
| Warm-Up | Pass out worksheet titled <br> Lesson Terminology and put <br> the matching transparency on <br> the overhead. | Please come in sit down and <br> begin filling out the paper on <br> your desk. You may use your <br> book, and some of these terms <br> may seem familiar to you. |
| Allow the students 5-7 <br> minutes to fill in their 3 <br> words. | N/A <br> Be sure you also come up with <br> an example or a picture that <br> words. (3-5 minutes) | represents your description. <br> This should help you in <br> remembering what the words <br> mean. |
| Engagement | Today we are going to focus <br> on some properties of <br> triangles. | Ask students for real-world |
| Applications of triangles. <br> aple | To begin, raise your hand if <br> you would like to share an <br> example of a triangle outside <br> of the classroom. | Varied student answers. |
| After hearing some examples <br> from students place the <br> transparency of real-world <br> triangle examples on the <br> overhead. | Here are some examples I <br> thought of, we have an electrical <br> grid tower, trusses on a roof, a <br> pennant, pyramids, a Dorito, the <br> sails on a sail boat, a triangular <br> shaped house, and another type <br> of truss used in sound and stage | 5 minutes |

Tie together the terms on the front of the paper with the real-world images on the back.

Have a class discussion pertaining to what the students found.

Triangle Sum Conjecture
Inform the class what we will be doing.

Instruct the class on the next task. Place the transparency with the step by step directions and example on the overhead. Pass out recording table with instructions.

Pass out protractors.

Pass out a set of six colored triangles to each group.
equipment.
At this time please find and at least one of each type of triangle listed on the front side of the paper. (An acute, a right and an obtuse triangle.) I would like for you to outline or shade it in and classify which type of triangle it is.
In real life, outside of this class do you walk around and see angles marked as ninety degrees, or that they are congruent? We must use reasonable judgment.

Raise your hand to share with the class which picture you chose for each type of triangle.

Now we will look at the angle measures of triangles.

For this activity you will be in groups of three so those tables not in threes already, please move to make a group of three. I am going to pass out a set of six colored triangles to your group. Each person is going to measure their two triangles using a protractor and record the angle measure in the recording table. You will then switch colors with a partner and measure those new angles. Record them in the table. Compare your angle measure with the angle measures your partner found for that triangle. Now on the half sheet of paper you are going to fill in the table and make a conjecture.

Who can share with the class what we are going to be doing?

And what are the possible types

Varied student answers.

15 minutes
But how can we assume it is a right angle?

| Wander around the classroom making sure all students are participating. <br> Get the students' attention back at the front of the classroom. Discuss the student findings. | of triangles we can have? <br> Raise your hand if you need a protractor. | Acute, Right, Obtuse. <br> All the triangles summed to $180^{\circ}$. There were two examples of each type of triangle, acute, obtuse, and right. |
| :---: | :---: | :---: |
|  | Be sure you are discussing your findings with all members of your group. |  |
|  | Raise your hand to share with the class some things you noticed. |  |
|  | Did the type of triangle matter? Why or why not? | No, no matter the type of triangle, the three angle measures will always sum to $180^{\circ}$. |
|  | Anything else anyone wants to add? |  |
| Instruct the students on the next step of the investigation. | Now what you're going to do is switch colors one last time, you should have the one color you haven't had yet. Carefully tear |  |
| Place the example transparency on the overhead. | off the three angles of the triangle and arrange them so that their vertices meet at one point. |  |
| Discuss the Triangle Sum Conjecture. | How does this arrangement verify the angle sum you found initially? | When we put all three vertices together we see that the pieces form a straight line. We know that a straight line measures $180^{\circ}$. |
|  | Is this a linear pair? Why or why not? | No, because a linear pair is composed of a pair, two, angles and this is composed of three. |
|  | Is this supplementary? Why or why not? | No, because once again, supplementary angles are composed of two angles, we have three. |
| Formally state what the Triangle Sum Conjecture is. | So knowing this, who wants to state in words what they think the Triangle Sum Conjecture might be? | The sum of the measures of the angles in every triangle is $180^{\circ}$. |

Now we are going to move on to discuss properties of isosceles triangles.

Place the Isosceles Triangle transparency on the overhead.

Pass out the handout with the 3 isosceles triangles on it.

Question students on parts of an isosceles triangle.

Instruct students to measure the missing angles. Discuss the findings.

Please raise your hand if you would like to define what an isosceles triangle is?

Just from looking at these triangles what do we know?
What do you notice?
What does that tell us?

Okay, you said they all sum to $180^{\circ}$, and what conjecture tells us that?

What are the names of the two missing angles?

What is the name of the angle that was given to you? How do you know that is the vertex angle?

As a group, measure the missing angles on the paper. What do you notice? (They are congruent.)

Did you expect that? Why or why not?

Could we have a triangle with two right angles? Why or why not?

But we could have a triangle with more than one obtuse angle, right? (Not really!) Why or why not?

An isosceles triangle is a triangle with at least two congruent sides. The angle between the congruent sides is called the vertex angle. The other two angles are called the base angles. The side between the base angles is called the base. The other two sides are called the legs.

We have three different types of triangles, acute, right and obtuse, and they all three have two congruent sides. They all sum to $180^{\circ}$.

## Triangle Sum Conjecture.

The base angles.

The vertex angle. It is the angle between the two congruent sides.

Varied student answers.

Varied student answers.
No, because then our other angle would have to be $0^{\circ}$ by the triangle Sum Conjecture and we wouldn't have a triangle.

No, because then the sum of our three angles would be great than $180^{\circ}$. (Similar to the previous question.)

| Discuss the angles found. | So we have $27^{\circ}, 27^{\circ}, 45^{\circ}, 45^{\circ}, 69^{\circ}$ and $69^{\circ}$. Think about these angle measures. What is common between the base angles on all three triangles? <br> Why is that? | They are all acute. <br> If they were right or obtuse our three angles would sum to more than $180^{\circ}$. Therefore, the two congruent angles must be acute. |
| :---: | :---: | :---: |
| Formally state what the Isosceles Triangle Conjecture is. | What can we conclude from this investigation? | That no matter whether the triangle is acute, right or obtuse, if it is isosceles, then its base angles are congruent. <br> If a triangle is isosceles, then its base angles are congruent. |
|  | Based on that conjecture, is an equilateral triangle an isosceles triangle? Why or why not? | Yes, because an equilateral triangle has three congruent angles, thus its two base angles are congruent. |
| Formally state what the Converse of the Isosceles Triangle Conjecture is. | So what would the converse of this conjecture be? Well first off, what is a converse? | With an if-then statement we check to see if the statement holds both forwards and backwards. |
|  | Is the converse true? What would the converse statement be? | Varied student answers. If a triangle has two congruent angles, then it is an isosceles triangle. |
|  | If a triangle has two congruent angles, does that automatically make it an isosceles triangle? Why do you say that? | Yes/No. Varied student explanations. |
|  | How could we check to be sure? | Varied student answers. |
|  | Well if we take our compass and measure the lengths of the sides, what are we checking for? | That the side lengths are the same length. |


| one of the isosceles triangles <br> from their paper on the overhead. | Thus, what have we found? | That the converse is true, if a <br> triangle has two congruent <br> angles, then it is an isosceles <br> triangle. |
| :--- | :--- | :--- |
| Wrap-Up |  |  |
| Your homework for tonight is <br> Section 4.1 \# 2-12 evens, 16, <br> $20-25$ |  |  |
| Section 4.2 \# 2-10 evens, 14- <br> 24 evens |  |  |
| Turn the class over to Dr. |  |  |
| Allison to release them to |  |  |
| lunch. |  |  |

6. Materials: Pencil, protractors (15), Discovering Geometry textbook, blank transparencies, printed transparencies, overhead markers, student worksheets, compass (1)
7. Resources*:
http://www.mathwarehouse.com/geometry/triangles/interactive-triangle.htm
http://www.mathopenref.com/isosceles.html
http://www.keymath.com/x19410.xml
8. Lesson Starter: See attached PDF. It is the pages from the teacher edition in their textbook (section $4.1 \& 4.2$ ).
(The student worksheets have a spot for the student's name, just not on here (below) because I copied and pasted these from the originals where the 'name' line is in the header. Also some of the worksheets and transparencies are in landscape format but to include everything in this one document I changed them to portrait.)

## Lesson 4.1 \& 4.2 Terminology

| Term | Description | Example/Picture |
| :---: | :---: | :---: |
| Acute |  |  |
| Triangle |  |  |
| Right |  |  |
| Triangle |  |  |
| Obtuse |  |  |
| Triangle |  |  |



1. Measure each angle of the two triangles of the same color.
2. Write the angle measures in the recording table.
3. Switch colors with another person. Measure and record those angles. Compare your angle measures with your partner's by looking at the recording table.
4. On the half sheet, classify each triangle as an acute, right or obtuse triangle, write the sum of the three angles and make a conjecture.


|  | Type of <br> Triangle | Sum of the <br> Angles |
| :---: | :---: | :---: |
| Triangle <br> 1 | Acute | $180^{\circ}$ |

## Conjecture:

## Group Recording Sheet

| Group <br> Member <br> Names |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Triangle 1's Angle Measures | $\frac{\circ}{\frac{m}{c} \angle a}$ | $\frac{\circ}{m \angle b}$ | $\bar{\circ} \bar{\circ}$ |  |  |  | $\frac{\circ}{m \angle a}$ | $\frac{\circ}{m \angle b}$ | $\overline{\mathrm{m} \angle c}$ |
| Triangle 2's <br> Angle <br> Measures |  | $\frac{\circ}{\substack{m \\ c}}$ | $\frac{\circ}{m<}$ |  |  |  | $\frac{\circ}{m \angle a}$ | $\frac{\circ}{m \angle b}$ | $\bar{\circ} \frac{\circ}{m \angle c}$ |
| Triangle 3's <br> Angle <br> Measures | $\frac{0}{m<a}$ |  | $\frac{\circ}{m \angle}$ |  |  |  |  | $\frac{\circ}{m \angle b}$ | $\overline{\mathrm{m} \angle c}$ |
| Triangle 4's Angle Measures | $\frac{0}{\frac{m}{c} \angle a}$ |  | $\bar{\circ}$ |  |  |  |  | $\bar{\circ} \frac{\circ}{m \angle b}$ | $\bar{\circ} \frac{\circ}{m \angle c}$ |
| Triangle 5's Angle Measures |  |  | $\frac{\circ}{m<}$ |  |  |  |  |  | $\frac{\circ}{m \angle c}$ |
| Triangle 6's Angle Measures | $\frac{\circ}{\frac{m}{c} \angle a}$ | $\bar{\circ} \overline{\mathrm{m} \angle b}$ | $\frac{\circ}{m \angle}$ |  | $\frac{\circ}{m \angle b}$ |  | $\frac{\circ}{m \angle a}$ | $\frac{\circ}{m \angle b}$ | $\bar{\circ} \frac{\circ}{m \angle c}$ |

Fill in the table below and make a conjecture from what you notice.

|  | Type of Triangle | Sum of the Angles |
| :--- | :--- | :--- |
| Triangle 1 |  |  |
| Triangle 2 |  |  |
| Triangle 3 |  |  |
| Triangle 4 |  |  |
| Triangle 5 |  |  |
| Triangle 6 |  |  |

Conjecture:

## The Triangle Sum



## How does this arrangement verify the angle sum you found initially?

## Isosceles Triangle



# Isosceles Triangle 



Isosceles Triangle Conjecture -

Picture:

Converse of the Isosceles Triangle Conjecture -

Picture:


## Lesson 2 Analysis

## Triangle Sum Conjecture

\&

## Properties of Isosceles Triangles



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December 2, 2010
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Classroom Interactions

## Analysis of Lesson 2

Some changes I implemented after viewing my first lesson and also after receiving feedback from my peer was to engage the class in more of a discussion or conversation about the questions and answers pertaining to the math instead of straight question and answer. This proved to be very beneficial; I feel that by having more of an open discussion the students were more able to express their opinions and concerns about the material. This allowed for further clarification on things that were not necessarily in my lesson plan to go over, for example when Josh asked if an equilateral triangle would be an isosceles triangle at the end of the lesson.

Another thing I made sure to change was not to talk over the students if they were talking while I was trying to say something. There were multiple times throughout the lesson that I told the class to be quiet and to give me their attention. This ensured that each student would be able to hear what myself or another student was saying. I found that in implementing this technique the students had more respect for me. They understood that I was not joking around and providing them with a free day.

My objectives for the lesson were:

- Students will be able to explain that the sum of the measures of the angles of a triangle is always $180^{\circ}$.
- Students will be able to describe a relationship related to the base angles of an isosceles triangle and make a conjecture about triangles that have two congruent angles.
- Students will be able to show problem solving skills and inductive reasoning skills as well as the implementation of new vocabulary.

The lesson objectives were communicated to the students throughout the different aspects of the lesson. The Triangle Sum Conjecture was addressed by having the students measure the angles of different types of triangles and find the sum. They then tore each triangle and saw that the angle measures combined to form a straight line which they already knew is $180^{\circ}$. We constructed the Triangle Sum Conjecture as a class. The second objective was taught through exploration of three different triangles. The students found that all three had congruent base angles and they also had congruent legs. From this we constructed the Isosceles Triangle Conjecture as a class. The third objective was communicated to the students through the review of the vocabulary terms and by using inductive reasoning to find our missing angles.

The first two objectives are primarily procedurally based, while the third objective is conceptual. Of the student artifacts, the homework problems require that the students apply the concepts in their computational work, while the class discussion, which can be seen below, captures more of the conceptual knowledge.

The students' understanding of each objective can be examined in the transcript (below) under the respective sections. Another artifact used in analyzing student's understanding is the student homework which was collected. The first objective is applied in section 4.1 problems 3 and 5. The second objective is applied in section 4.2 problems 1-9. The third objective is applied throughout the class discussion and can be viewed periodically throughout the video, more specifically, 30 minutes into the video. In reviewing the students' homework, ten of the twelve students who turned in homework completed both questions 3 and 5 correctly in section
4.1, demonstrating an understanding of the Triangle Sum Conjecture. Kaitlyn got number 3 correct, but number 5 was incorrect. Looking at her work it appears that it was a simple calculation error. Juan incorrectly answered both numbers 3 and 5. He probably did not understand the Triangle Sum Conjecture. For the second objective, questions 1-9 test for an understanding of the isosceles conjecture and its converse. Three quarters of the class got numbers 1 and 5 correct. Both of these questions demonstrate knowledge of the objective. In analyzing many of the other problems from this section it is apparent that the students knew what they were doing, they just made computational mistakes, or sometimes solved for the wrong variable. The correct thought process was there. For the third objective, demonstration of problem solving skills was tested in questions 7 and 9. After looking at the student homework, none of the students completed both of these problems fully correct, however, seventy percent of the class got half of question 7 correct and all but one student got half of question number 9 correct. Each of these questions consisted of two parts and many times, as in questions 1 and 5, the students made simple computational errors and solved for the wrong part.

The questioning strategies will be analyzed through the video recording taken during the lesson. The teacher questions and student answers have been transcribed. By reading and analyzing the questions asked by the teacher and the answers given by the students I will be able to make note of the type of question asked and the value of the answer given in terms of the amount of student knowledge the teacher is presented with. Each question will be classified as one of the following four types of questions and will be denoted at the beginning of the question. Following each student answer will be a number from 0 to 2 which will represent the amount of student understanding the teacher received from the student or the whole class in response to the presented question. A response tagged with a 0 means that the teacher gained no knowledge of the student's understanding of the content. A 1 means that the teacher gained some information but strongly believes if the student had been pressed harder that they would have gotten more information. A 2 means that the answer given satisfies the question asked and the teacher is content with the amount of knowledge displayed.

Controlling question - A question that seems to control or sway the student's answer. This does not reveal much about the student's level of understanding, or misunderstanding.

Closed form question - A question that seeks a particular answer, right or wrong, true or false.
Open form question- These questions include a how and/or a why, their goal is to promote a description of a strategy or method used.

Challenge question - A question that follows after a student has given an answer to the initial question. This challenge question requires the student to elaborate and explain how they found that answer. Not just accepting a correct answer and assuming that the student used proper reasoning to arrive at that answer.

## Transcript with coding -

5:20-9:45 Questioning begins when we review as a class what they found for their descriptions and pictures of: acute, right and obtuse triangles.
[OPEN] Me - Who would like to share with the class their description of acute angle? Kaitlin. Kaitlin - A triangle that has a measurement of $180^{\circ}$. The angles. [1]
[CHALLENGE] Me - Which angles? How many angles?
Whole class - All 3. [1]
[CHALLENGE] Me - All 3? What do you guys think?
Whole class - At least one. At least two. [1]
[CHLLENGE] Me - One, two three?
Me - Okay well the description I have for acute triangle is where all three interior angles are less than $90^{\circ}$.
[OPEN] Me - Who wants to describe right triangles? Jacob.
Jacob - A triangle that has one $90^{\circ}$ angle. [1]
[CHALLENGE] $M e$ - Is it at least one $90^{\circ}$ angle?
Jacob - At most. It can't have more than one. [1]
[CHALLENGE] Me - What would be wrong if it had two $90^{\circ}$ angles?
Jacob - It would be a square. [1]
Student - All three angle measures have to add up to $180^{\circ}$. [1]
Me- Repeat above quote, we would have an angle that measures $90^{\circ}$, another angle measure of $90^{\circ}$ and that adds up to... $180^{\circ}$. Sienna.

Sienna - You need all three angles to add up to $180^{\circ}$, but if two of them are $90^{\circ}$ that adds up to $180^{\circ}$. [2]
[CLOSED] $M e$ - And then our third angle would be what?
Sienna - Nothing. [1]
[CLOSED] Me - Zero, so that's what shape? Jacob.
Jacob - A straight line. [2]
[OPEN] Me - Okay, description for obtuse triangle? Tristian.

Tristian - A triangle with one obtuse angle. [2]
[CLOSED] Me - Repeat above quote. Does anybody have anything different? No? Do you guys agree with that?
[CLOSED] $M e$ - Called on a random student, do you agree with our description?
Student - If it has more than one obtuse angle, then the sum of the three angles would be greater than $180^{\circ}$. [2]
[CLOSED] $M e$ - Are there any questions about this paper?
11:49-13:30 Questioning about labeling real-world triangles.
[CLOSED] Me - Raise your hand to tell me which one you found is an acute triangle?
Student - The gator pennant. [1]
[CHALLENGE] Me - Why do you say that? Because you can look at it and say that I can tell that all three of these are less than $90^{\circ}$. Are there any other acutes? Geneva.

Geneva - The Dorito. [1]
[CONTROLLING] $M e$ - For the same reason, right?
[OPEN] Me - Okay, what about the right triangle? Jeremy.
Jeremy - The sailboat. [1]
[OPEN] Me - Now what about an obtuse triangle? George.
George - The framework. [1]
[CHALLENGE] - Me - Ok, which angle? (labeled them a, b, c.)
George - A. [2]
$\mathbf{2 5 : 3 0} \mathbf{- 2 7 : 2 2}$ Questioning about cooperative group activity.
$M e$ - Now what we are going to do is talk about some characteristics of triangles. (told students what we are going to be doing.)
$M e$ - Alright guys can I have your attention? Put your protractors down, pencils down.
Um, let's talk about these triangles.
[CLOSED] Me - What kinds of different triangles did you have?
Student - Acute, obtuse and right. [2]
[OPEN] Me - Repeat above quote. Good, what did you notice about the angle measurements?
[CLOSED] Me - What did they sum to?

Whole class - $180^{\circ}$ ! [2]
[CHALLENGE] $M e$ - All of them or some of them?
Whole class - All of them! [1]
[CHALLENGE] $M e$ - And why is that?
Student - Because they are triangles. [1]
$M e-B u t ~ t h e y ~ a r e ~ d i f f e r e n t ~ t y p e s ~ o f ~ t r i a n g l e s . ~$
Students - A triangle is a triangle, all triangles sum to $180^{\circ}$ [2]
29:25-30:00 Discussion about tearing angles to find a line.
[CLOSED] $M e$ - Instructed students to rip the triangle papers.
Students - It equals $180^{\circ}$, it forms a straight line. [1]
30:23-31:00 Questioning about isosceles triangles.
$M e$ - Now we are going to move on and talk about isosceles triangles.
[OPEN] Me - Who can define what an isosceles triangle is?
Jacob - Isn't it where two sides of the triangle are congruent? [2]
$M e$ - Jacob said an isosceles triangle has two congruent sides. Now I want you to write your own definition on your paper.

34:15-37:25 Discussion about Isosceles and the Conjecture.
[CLOSED] Me - What did you guys put for here? (one of the base angles.)
Whole Class $-45^{\circ}$. [1]
[CLOSED] $M e$ - And what about here?
Whole Class - $45^{\circ}$. [1]
[CLOSED] $M e$ - So they are the same?
Whole Class - Yes. [1]
[CLOSED] $M e$ - Did you measure them with a protractor?
Whole Class - No. [1]
[CHALLENGE] Me - You didn't use a protractor? Then what did you do?
Jacob - You subtract from $180^{\circ}$ then divide by 2. [1]
[CHALLENGE] $M e$ - What did you subtract?

Jacob - The number given. [1]
[CHALLENGE] $M e$ - So what about our obtuse triangle?
Student - $180^{\circ}$ minus $126^{\circ}$ divided by 2. [2]
[CHALLENGE] $M e$ - Then how about our acute triangle? (same answers)
[OPEN] Me - What did they all have in common?
Student - They are all triangles. [1]
[OPEN] Me - Right, they are all triangles, what else do they all have in common?
Student - They are all isosceles triangles, they all equal $180^{\circ}$. [2]
[CONTROLLING] $M e$ - They are all isosceles triangles, do you see that?
[OPEN] Me - What did we notice about the base angles for each triangle?
Student - They are congruent. [2]
Student - Isosceles Triangle Conjecture - If a triangle is isosceles then its base angles are congruent. [2]
[CONTROLLING] $M e$ - Repeat above quote. Correct, and we see that right here right?
38:00-41:00 Discussion about converse.
[CLOSED] Me - The next one says the Converse of the Isosceles Triangle Conjecture. What is a converse? Zachary.

Zachary - If the base angles are congruent then it is isosceles. [2]
[OPEN] Me - Can you draw what the converse conjecture would look like on your paper?
42:05-44:40 Is an equilateral isosceles?
[CLOSED] $M e$ - Would an equilateral triangle be an isosceles triangle?
Whole class - No! [1]
[CHALLENGE] Me - No? Why not?
Students explaining that an isosceles need the 2 base angles to be congruent but an equilateral has 3 congruent angles. [2]
[OPEN] Me - What do we need to be an isosceles triangle?
Whole Class - Two congruent sides. [2]
[CLOSED] $M e$ - Right, and there would be two congruent angles. Does an equilateral triangle
have two congruent sides?
Whole Class - Yes/No. [1]
[CONTROLLING] $M e$ - Yes! It has three, so that means it has two, right? Okay, are there two congruent angles?

Student - Yeah. [1]
Adonna - But...
[OPEN] Me - But what?
Adonna - But there is another angle. [2]
[CLOSED] $M e$ - Draws an equilateral triangle on the overhead. And asks, are there two congruent angles in this picture?

Whole Class - Yes. [1]
Geneva - But an isosceles ONLY has two. [2]
[CLOSED] Me - Do I have the word only on here? (referring to the conjecture) It needs to have two and it does have two.
[CLOSED] Me - Do you guys have any questions? Are you guys clear that this does fit the description of an isosceles triangle?

I feel that my questions varied throughout the entire lesson. I had higher order questions spread throughout as well as recall questions every once and awhile. It was nice to see that as mentioned in the Manouchehri and Lapp article, I was sure to follow up on students' answers and ask for them to elaborate further. Another technique from the article I implemented into my lesson was to be flexible and be willing to add to or subtract aspects of your lesson depending on the flow of the class.

For the cooperative group work, I was pleasantly surprised at how smoothly it went. Once the class got into groups of three, I gave the directions and had an example of what they were to be doing on the overhead, and then many of the groups began working. I wandered around the room individually explaining the directions to a few students but after that they were fine. Each student had a task and a responsibility and they were all engaged, which implemented Johnson, Johnson and Holubec's suggestion of individual accountability. There was positive interdependence built into the activity because the angle measure found by one group member would then be verified by another. The students were checking each other to be sure they agreed on the angle measures. The activity included face-to-face promotive interaction because the students all shared the same recording sheet and were able to compare their results with their
group member's. The students were encouraged to challenge their group member if they found a different angle measure. In this activity, I feel that I allowed for group processing by providing sufficient time to pass by allowing the class about ten minutes to work together.

Most all of the class participated in the class discussion at one point or another. There were some questions that I asked where the entire class answered. If while walking around the room I noticed that a specific student wasn't working I was sure to call on them during the class discussion. Both males and females answered specific questions in the discussions, as well as a student with special needs. I am not aware that there are any English Language Learners in the class. The entire class participated in the activities. This can be seen in the video recording.

The lesson provided accommodations to both English Language Learners as well as students with disabilities. Throughout the lesson I specifically addressed the technical math vocabulary that we would be using throughout the lesson. Also, I was sure to pay special attention that George was on task and following instructions and not "playing on his computer" as Dr. Allison had advised. I could improve in the accommodation area though, by making sure I write down many, if not all, of the important statements that are mentioned. This would make it visual for each student and probably avoid myself from having to repeat things.

My two main forms of formative assessment throughout the lesson were through questioning and looking at the students' work on their papers. This helped me to determine how much time I needed to spend on each part as we went through the lesson. If I was asking questions and all the students already knew what question was coming next, they were ready to move on. However, if they were providing some incorrect answers or were not definite about the topic I would question deeper. A specific case where this happened was when a question came up that wasn't in my lesson plan. "Are equilateral triangles isosceles triangles?" I presented the entire class with this question which in turn sparked a five minute debate. (Time marker) It was fantastic. Many of the students felt that an equilateral triangle couldn't be an isosceles triangle. Myself and a few other students questioned them. Eventually, through continuous questioning and inductive reasoning we came to conclude that equilateral triangles could be, in fact are, isosceles triangles. A case where the students assessed their own learning was when they had to measure the angle measure of the different types of triangles. They had to use their knowledge of the Triangle Sum Conjecture to be sure that the numbers they were finding fit properly.

I feel that the most important way to ensure that the students gain a complete understanding of my three objectives is through the discovery learning they did for both the Triangle Sum Conjecture and the Isosceles Triangle Conjecture.

I feel that the most crucial aspect to the students understanding the lesson objectives was the class discussion lead by the teacher questions. Through class discussions the teacher is able to gauge where the students' understanding is and can make a decision about whether to move on or to spend more time on a specific topic. In the article by Breyfogle and Herbel-Eisenmann,
they address how important it is that the teacher has an understanding of the students' understanding. With this form of open class discussion the students are allowed to think more outside the box instead of the streamlined lesson the teacher has typed up. Being able to wander down certain roads and to explore a little further has tremendous benefits. One aspect mentioned in the article that I failed to do was allow an appropriate amount of wait time after asking each question. If I had though, I probably would have had a greater number of students willing to answer questions with more thought out answers.

In the future if I were to teach this lesson again, or revise it for anyone else to teach, the aspect I would want to change the most is the cooperative group activity. Going off of the Johnson, Johnson, and Holubec article on group work, I would like to alter it so that it entailed more individual responsibility and then held each group responsible for their information, requiring each group member to check that each member of the group knows what's going on and how to complete each part of the activity. I would like to also give the groups different triangles so that they could present their findings to the class. With these changes I think the lesson would be more structured and flow more smoothly.

For my self-evaluation of my Lesson 2 Analysis, I would give myself the following scores: Presentation - 3, I feel that I have a nicely presented paper with the supporting material included. Evidence of students having met learning objectives - 3, By using two forms of student artifacts, and providing reasoning for how and why I believe the students answered each question it showed me which objectives they mastered. Revisions for future - 3, Using research, I have listed some changes I would make to the lesson. Rubric - 3, I have attached my selfevaluation to the back of my paper.

