LESSON 2.5

PLANNING

LESSON OUTLINE

First day:

30 min Investigation 1

15 min Investigation 2

Second day:

15 min Investigation 2

15 min Sharing

5 min Closing

10 min Exercises

MATERIALS

- protractors
- straightedges
- patty paper or tracing paper
- Vertical Angles (W), *optional*
- Sketchpad demonstration Linear Pairs and Vertical Angles, *optional*
- Sketchpad activity Angle Relationships, *optional*

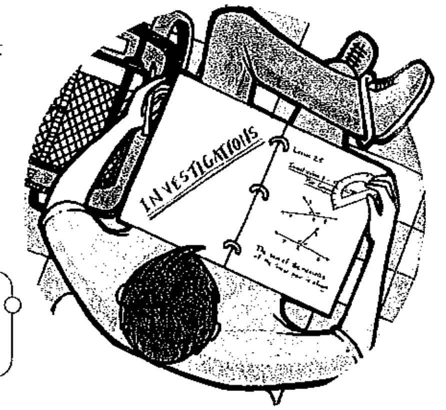
LESSON 2.5

Angle Relationships

Discovery consists of looking at the same thing as everyone else and thinking something different.

ALBERT SZENT-GYÖRGYI

Now that you've had experience with inductive reasoning, let's use it to start discovering geometric relationships. This investigation is the first of many investigations you will do using your geometry tools.

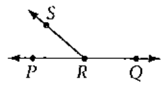


Create an investigation section in your notebook. Include a title and illustration for each investigation and write a statement summarizing the results of each one.

Investigation 1
The Linear Pair Conjecture

You will need

- a protractor

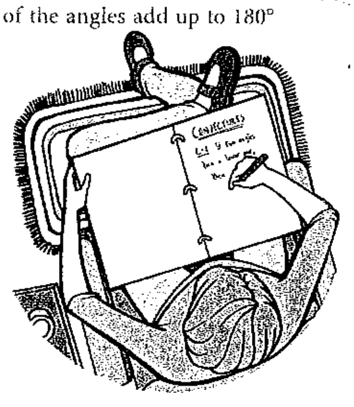


Step 1 On a sheet of paper, draw $\overleftrightarrow{PQ}$ and place a point R between P and Q. Choose another point S not on $\overleftrightarrow{PQ}$ and draw $\overrightarrow{RS}$. You have just created a linear pair of angles. Place the "zero edge" of your protractor along $\overleftrightarrow{PQ}$. What do you notice about the sum of the measures of the linear pair of angles? 180°

Step 2 Compare your results with those of your group. Does everyone make the same observation? Complete the statement.

Linear Pair Conjecture C-1

If two angles form a linear pair, then __?__. the measures of the angles add up to 180°



The important conjectures have been given a name and a number. Start a list of them in your notebook. The Linear Pair Conjecture (C-1) and the Vertical Angles Conjecture (C-2) should be the first entries on your list. Make a sketch for each conjecture.

TEACHING

You can start with the one-step investigation below or begin by reviewing the definitions of *complementary, supplementary, vertical,* and *linear pairs of angles* before proceeding to the investigations. **[Alert]** Students may think that linear pairs of angles are always horizontally oriented. Use a variety of orientations when you illustrate geometry terms.

Guiding Investigation 1

You might use this investigation as a follow-along activity or use the Sketchpad demonstration Linear Pairs and Vertical Angles.

Step 1 Remind students that they can extend the rays to measure the angles. Conjectures may include that the angles are supplementary, that the sum of their measures is 180°, that the angles are adjacent, that they "make a line" or, as Euclid put it, "together form two right angles." Only one conjecture is intended but be open to all conjectures at this point.

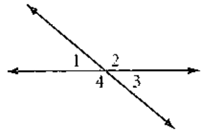
One Step

Draw two intersecting lines, point out a pair of vertical angles and a linear pair of angles, and ask how their sizes are related. Accept all conjectures without judgment, and ask students to investigate in their groups by drawing pairs of intersecting lines and measuring the angles formed.

LESSON OBJECTIVES

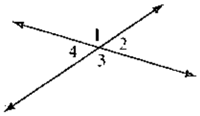
- Discover relationships between special pairs of angles
- Practice performing investigations and writing conjectures
- Practice measurement skills
- Develop inductive and deductive reasoning and cooperative behavior

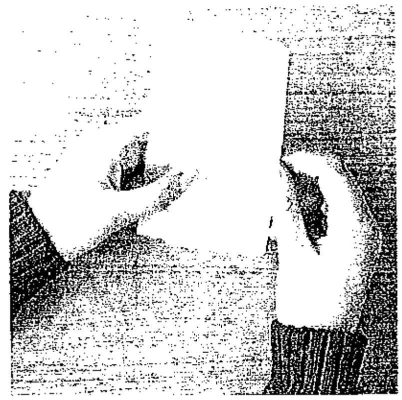
In the previous investigation you discovered the relationship between a linear pair of angles, such as ∠1 and ∠2 in the diagram at right. You will discover the relationship between vertical angles, such as ∠1 and ∠3, in the next investigation.



Investigation 2
Vertical Angles Conjecture

You will need
- a straightedge
- patty paper





Step 1 ∠1 and ∠3; ∠2 and ∠4

Step 1 Draw two intersecting lines onto patty paper or tracing paper. Label the angles as shown. Which angles are vertical angles?

Step 2 They are equal.

Step 2 Fold the paper so that the vertical angles lie over each other. What do you notice about their measures?

Step 3 They are equal

Step 3 Fold the paper so that the other pair of vertical angles lie over each other. What do you notice about their measures?

Step 4 Compare your results with the results of others. Complete the statement.

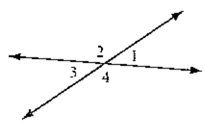
Vertical Angles Conjecture C-2

If two angles are vertical angles, then _?_. they have equal measures (are congruent)

Developing Proof You used inductive reasoning to discover both the Linear Pair Conjecture and the Vertical Angles Conjecture. Are they related? If you accept the Linear Pair Conjecture as true, can you use deductive reasoning to show that the Vertical Angles Conjecture must be true?

Read the example below. Without turning the page, write a deductive argument with your group. Remember the reasoning strategy of representing a situation algebraically. Another strategy is to apply previous conjectures and definitions to a new situation. Then compare your solution to the one on the next page.

EXAMPLE Use the Linear Pair Conjecture and the diagram at right to write a deductive argument explaining why ∠1 must be congruent to ∠3.



NCTM STANDARDS

CONTENT		PROCESS	
	Number		Problem Solving
	Algebra	✓	Reasoning
✓	Geometry	✓	Communication
✓	Measurement		Connections
	Data/Probability		Representation

▶ *EXAMPLE*

Students may have difficulty following the reasoning in this proof. Encourage them to take one step at a time, but don't assume that if they follow each step then they understand the proof. Have them give the reasoning in their own words. They might say something like "Adding the same thing to these two angles gives the same result, 180°, so these two angles must be equal." Push for precision in the use of terms only if students are comfortable with the logic.

One Step (continued)
As you circulate, ask groups to write conjectures beginning with "If two angles are vertical angles, then . . . ." During Sharing, ask what would be needed to give a deductive explanation of the Vertical Angles Conjecture. Students may have several suggestions. Try to be sure that included among them is an explanation referring to the Linear Pair Conjecture.

Guiding Investigation 2

If students did the first investigation and time is short, you might do this investigation as a class.

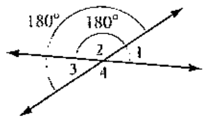
Protractors aren't really needed, because folding is sufficient, but students might prefer to measure. Reasonable conjectures might be that vertical angles have the same measure, that they are congruent, or that they are symmetric about a line. Welcome all conjectures.

Ask students to attach (glue, tape, or staple) the patty paper to their reports in the investigation section of their notebooks. (Using a small amount of removable glue allows you to pull the patty paper off later to check how the investigation was done and then re-attach it to the notebook.)

Developing Proof
Students may struggle to write a deductive argument. Encourage them to draw a diagram and mark it with what they know. [Ask] "If you know one of the angle measures, can you find the others without measuring?" [Yes, using the Linear Pair Conjecture, all the other angle measures can be determined.]

You might hand out the worksheet Vertical Angles and ask students to close their books so they aren't tempted to look at the solution before they have attempted the proof themselves.

▷ Solution



You can see from the diagram that the sum of the measures of angles 1 and 2 is equal to the sum of the measures of angles 2 and 3 because they are both linear pairs. Because angle 2 is the same in both sums, angle 1 must equal angle 3. To write a deductive argument, go through this logic one step at a time.

Deductive Argument

For any linear pair of angles, their measures add up to 180°.

$m\angle 1 + m\angle 2 = 180°$

$m\angle 2 + m\angle 3 = 180°$

Since both expressions on the left equal 180°, they equal each other.

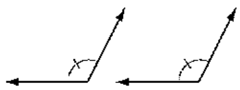
$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$

Subtract $m\angle 2$ from both sides of the equation.

$m\angle 1 = m\angle 3$

Vertical angles 1 and 3 have equal measures, so they are congruent. ■

You discovered the Vertical Angles Conjecture: If two angles are vertical angles, then they are congruent. Does that also mean that all congruent angles are vertical angles? The **converse** of an "if-then" statement switches the "if" and "then" parts. The converse of the Vertical Angles Conjecture may be stated: If two angles are congruent, then they are vertical angles. Is this converse statement true? Remembe that if you can find even one counterexample, like the diagram below, then the statement is false.



Therefore, the converse of the Vertical Angles Conjecture is false.

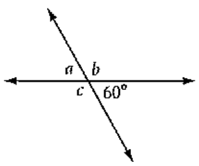
EXERCISES

You will need

Geometry software for Exercise 12

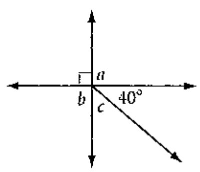
Without using a protractor, but with the aid of your two new conjectures, find the measure of each lettered angle in Exercises 1–5. Copy the diagrams so that you can write on them. List your answers in alphabetical order.

1.



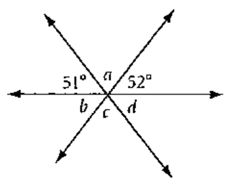
$a = 60°, b = 120°, c = 120°$

2.



$a = 90°, b = 90°, c = 50°$

3.



$a = 77°, b = 52°, c = 77°, d = 5$

SHARING IDEAS

Pick a variety of conjectures to be shared. As a class, discuss their mathematical value.
[Ask] "Do the conjectures capture any insight into the situation?" "Will they be useful?" Point out that in the future you'll be referring back to formal conjectures, so you need to agree on what is meant by each. If students want to have several competing conjectures be "official," you might give different names to them.

Allow disagreement. As students defend their ideas, they clarify their thinking. Some initial confusion can motivate deeper learning, but avoid allowing students to feel a debilitating degree of frustration.

[Ask] "What kind of reasoning led to the conjectures?" [Because the conjectures were based on patterns detected in several measurements, they were arrived at inductively.] In a mathematical sense, conjectures are not accepted as true until they have been proved deductively. Once a conjecture has been proved deductively, it is a theorem. In this course, conclusions arrived at inductively are assumed to be true. They will not be called *theorems* until they are proved within a mathematical system in Chapter 13.

If the idea hasn't already come up, have students trace the lines onto patty paper and then rotate the patty paper 180° to place each angle on top of its vertical angle. **[Ask]** "Is this a deductive explanation of the Vertical Angles Conjecture?" [yes, with the understanding that the angle measures do not change when rotated]

Assessing Progress

As you observe students working and presenting their ideas, you can assess their ability to reason inductively, that is, to collect good data, look for patterns, and write a conjecture. In addition, you can check how well they use straightedges and protractors and assess their social skills during group work.

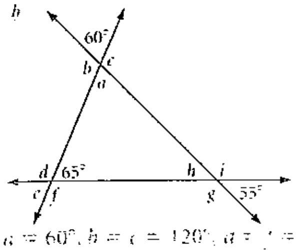
Converse

Insight into a theorem can often be gained by considering its converse. Besides considering the converse of the Vertical Angles Conjecture, students will look at the converse of the Linear Pair Conjecture in Exercise 9.

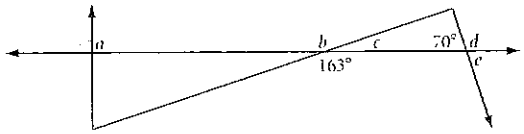
Closing the Lesson

Remind students that inductive reasoning led to conjectures about linear pairs of angles and pairs o vertical angles. Restate the agreed-on Linear Pair Conjecture and Vertical Angles Conjecture and any other important conjectures that arose. The Vertica Angles Conjecture was proved deductively.

If students seem to be having difficulty with the conjectures, you might work one of the first five exercises together.

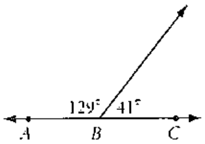
4. 

$a = 60°$, $b = c = 120°$, $d = f = 115°$,
$e = 65°$, $g = i = 125°$, $h = 55°$

5. 

$a = 90°$, $b = 163°$, $c = 17°$, $d = 110°$, $e = 70°$

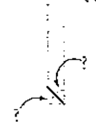
6. *Developing Proof* Points A, B, and C at right are collinear. What's wrong with this picture?



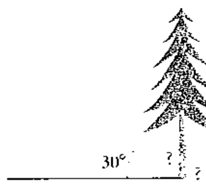
The measures of the linear pair of angles add up to 170°, not 180°.

7. Yoshi is building a cold frame for his plants. He wants to cut two wood strips so that they'll fit together to make a right-angled corner. At what angle should he cut ends of the strips?

The angles at which he should cut measure 45°.



8. A tree on a 30° slope grows straight up. What are the measures of the greatest and smallest angles the tree makes with the hill? Explain.



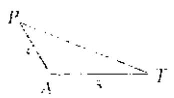
9. You discovered that if a pair of angles is a linear pair, then the angles are supplementary. Does that mean that all supplementary angles form a linear pair of angles? Is the converse true? If not, sketch a counterexample.
The converse is not true.

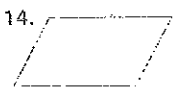
10. If two congruent angles are supplementary, what must be true of the two angles? Make a sketch, then complete the following conjecture: If two angles are both congruent and supplementary, then __?__. each must be a right angle

11. *Developing Proof* Using algebra, write a deductive argument that explains why the conjecture from Exercise 10 is true. Let the measures of the congruent angles be x. They are supplementary, so $x + x = 180°$, $2x = 180°$, $x = 90°$. Thus each angle is a right angle.

12. *Technology* Use geometry software to construct two intersecting lines. Measure a pair of vertical angles. Use the software to calculate the ratio of their measures. What is the ratio? Drag one of the lines. Does the ratio ever change? Does this demonstration convince you that the Vertical Angles Conjecture is true? Does it explain why it is true? The ratio is 1. The ratio does not change as long as the lines don't coincide. Because the demonstration does not explain why, it is not a proof.

Review

13. 

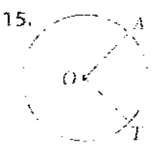
14. 

For Exercises 13–17, sketch, label, and mark the figure.

1.5 **13.** Scalene obtuse triangle PAT with $PA = 3$ cm, $AT = 5$ cm, and $\angle A$ an obtuse angle

1.6 **14.** A quadrilateral that has rotational symmetry, but not reflectional symmetry

1.7 **15.** A circle with center at O and radii $\overline{OA}$ and $\overline{OT}$ creating a minor arc $\overset{\frown}{AT}$

15. 

Exercise 10 This is the Congruent Supplements Conjecture; it will be a useful shortcut in some later proofs.

BUILDING UNDERSTANDING

The exercises provide applications of the two conjectures from the investigations.

ASSIGNING HOMEWORK

Essential	1–10
Performance assessment	9
Journal	8, 11, 12
Group	4, 5, 11
Review	13–28
Algebra review	11, 22–27

MATERIALS

- Exercises 1–5 (T), *optional*

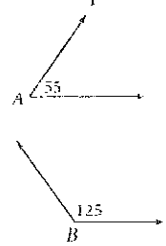
Helping with the Exercises

Exercise 6 You might want to model the kinds of answers students should be looking for in exercises such as this first of many instances of "What's wrong with this picture?"

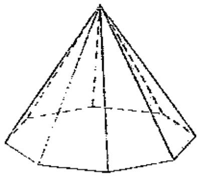
8. Greatest: 120°. Smallest: 60°. One possible explanation: The tree is perpendicular to the horizontal. The angle of the hill measures 30°. The smaller angle and the angle between the hill and the horizontal form a pair of complementary angles, so the smaller angle equals $90° - 30° = 60°$. The smaller angle and larger angle form a linear pair, so the larger angle equals $180° - 60° = 120°$.

Exercise 9 To defend the opinion that the converse is false, students need to give a counterexample.

9. sample counterexample:

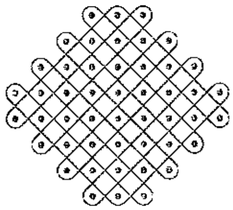


16.

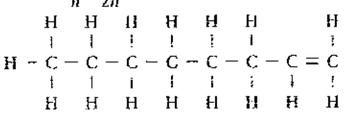


18. Possible answer: All the cards look exactly as they did, so it must be the 4 of diamonds, because it has rotational symmetry while the others do not.

20.



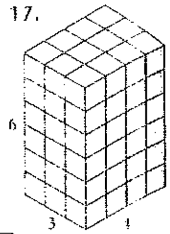
21. C_nH_{2n}



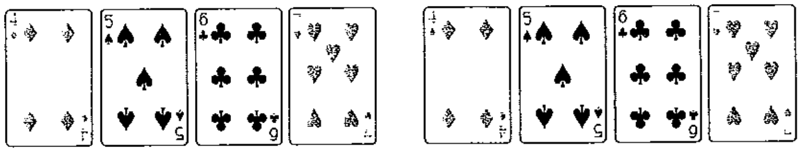
1.8 **16.** A pyramid with an octagonal base

1.8 **17.** A 3-by-4-by-6-inch rectangular solid rests on its smallest face. Draw lines on the three visible faces to show how you can divide it into 72 identical smaller cubes.

17.



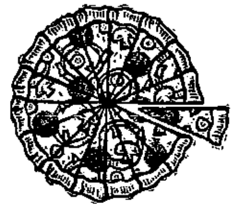
Chapter 0 **18.** Miriam the Magnificent placed four cards face up (the first four cards shown below). Blindfolded, she asked someone from her audience to come up to the stage and turn one card 180°.



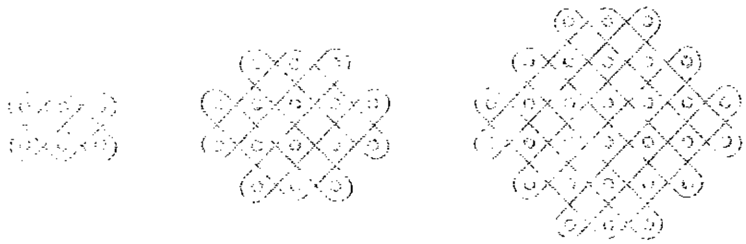
Before turn After turn

Miriam removed her blindfold and claimed she was able to determine which card was turned 180°. What is her trick? Can you figure out which card was turned? Explain.

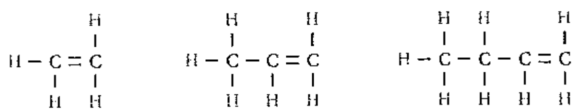
1.7 **19.** If a pizza is cut into 16 congruent pieces, how many degrees are in each angle at the center of the pizza? 22.5°



2.2 **20.** Paulus Gerdes, a mathematician from Mozambique, uses traditional *lusona* patterns from Angola to practice inductive thinking. Shown below are three *sona* designs. Sketch the fourth *sona* design, assuming the pattern continues.



2.2 **21.** Hydrocarbon molecules in which all the bonds between the carbon atoms are single bonds except one double bond are called *alkenes*. The first three alkenes are modeled below.



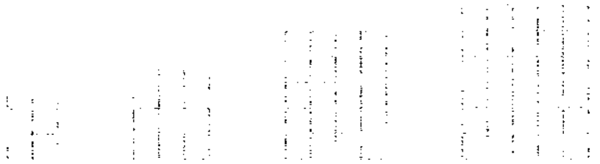
Ethene (C_2H_4) Propene (C_3H_6) Butene (C_4H_8)

Sketch the alkene with eight carbons in the chain. What is the general rule for alkenes ($C_nH_?$)? In other words, if there are n carbon atoms (C), how many hydrogen atoms (H) are in the alkene?

Science CONNECTION

Organic chemistry is the study of carbon compounds and their reactions. Drugs, vitamins, synthetic fibers, and food all contain organic molecules. To learn about new advances in organic chemistry, go to www.keymath.com/DG .

2.2 **22.** If the pattern of rectangles continues, what is the rule for the perimeter of the nth rectangle, and what is the perimeter of the 200th rectangle? What is the rule for the number of 1-by-1 squares in the nth rectangle, and how many 1-by-1 squares are in the 200th rectangle?



$4n + 6$

Rectangle	1	2	3	4	5	6	...	n	...	200
Perimeter of rectangle	10	14	18	22	26	30	...		...	806
Number of squares	6	12	20	30	42	56	...		...	

40,602

$(n + 1)(n + 2)$

2.3 **23.** The twelfth-grade class of 80 students is assembled in a large circle on the football field at halftime. Each student is connected by a string to each of the other class members. How many pieces of string are necessary to connect each student to all the others? handshake problem: $\frac{n(n-1)}{2}$; $\frac{80(79)}{2} = 3160$ pieces of string

Exercise 23 If students are struggling with this problem, remind them of the handshake problem.

2.3 **24.** If you draw 80 lines on a piece of paper so that no 2 lines are parallel to each other and no 3 lines pass through the same point, how many intersections will there be? 3160 intersections

2.3 **25.** If there are 20 couples at a party, how many different handshakes can there be between pairs of people? Assume that the two people in each couple do not shake hands with each other. $\frac{n(n-2)}{2}$ yields 760 handshakes.

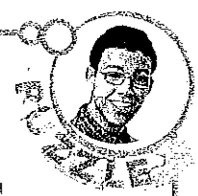
2.2 **26.** If a polygon has 24 sides, how many diagonals are there from each vertex? How many diagonals are there in all? 21; 252

2.3 **27.** If a polygon has a total of 560 diagonals, how many vertices does it have? $\frac{n(n-3)}{2} = 560$; there are 35 vertices.

Exercise 27 This problem may be solved using trial-and-error. Students can take a reasonable guess at the value for n and then check it by substituting it into the formula.

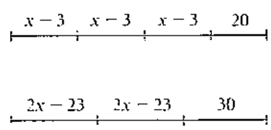
2.4 **28.** A midpoint divides a segment into two congruent segments. Point M divides segment $\overline{AY}$ into two congruent segments $\overline{AM}$ and $\overline{MY}$. What conclusion can you make? What type of reasoning did you use? M is the midpoint of $\overline{AY}$; deductive.

IMPROVING YOUR ALGEBRA SKILLS



Number Line Diagrams

1. The two segments at right have the same length. Translate the number line diagram into an equation, then solve for the variable x.



2. Translate this equation into a number line diagram.
$2(x + 3) + 14 = 3(x - 4) + 11$

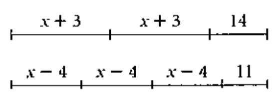
IMPROVING ALGEBRA SKILLS

[Ask] "What do you know about the two line segments?" [The segments are equal in length, although they are broken into pieces of different lengths.] "How could knowing this information help you set up an equation that could be solved?"

Note that these problems provide both algebraic and geometric representations of the same quantities.

1. $3(x - 3) + 20 = 2(2x - 23) + 30$; $x = 27$

2.



LESSON 2.6

PLANNING

LESSON OUTLINE

One day:

25 min Investigation

5 min Sharing

5 min Closing

10 min Exercises

MATERIALS

- lined paper
- straightedges
- patty paper
- protractors, *optional*
- Alternate Interior Angles (W), *optional*
- Sketchpad activity Special Angles on Parallel Lines, *optional*

TEACHING

For a quick review and informal assessment, you might ask each group to show a linear pair and vertical angles with materials at hand (such as pencils). Then begin the lesson with the one-step investigation or introduce the definitions at the beginning of the lesson.

You might choose to use the Sketchpad activity Special Angles on Parallel Lines.

One Step

Draw two parallel line segments and a segment intersecting them to introduce the term *transversal.* **[Ask]** "How many angles are formed, and what relationships are there among those angles?" As students work, encourage them to state their observations as conjectures in if-then form. Conjectures won't be clearly stated unless students know names for the angles, but you need not show agreement or disagreement with any conjecture at this point. Challenge any group that comes up with a conjecture to consider its converse, reminding students as needed how to form the converse. During Sharing, introduce the terminology for angles when students see that their conjectures need clarifying.

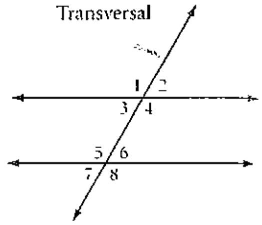
LESSON

2.6 Special Angles on Parallel Lines

The greatest mistake you can make in life is to be continually fearing that you will make one.

ELBERT HUBBARD

A line intersecting two or more other lines in the plane is called a **transversal.** A transversal creates different types of angle pairs. Three types are listed below.

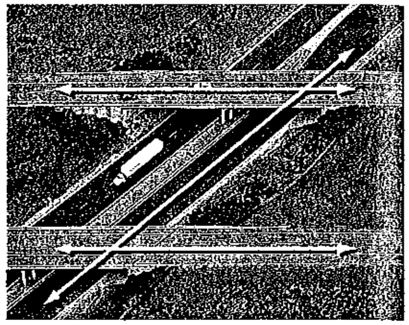


One pair of **corresponding angles** is ∠1 and ∠5. Can you find three more pairs of corresponding angles?

One pair of **alternate interior angles** is ∠3 and ∠6. Do you see another pair of alternate interior angles?

One pair of **alternate exterior angles** is ∠2 and ∠7. Do you see the other pair of alternate exterior angles?

When parallel lines are cut by a transversal, there is a special relationship among the angles. Let's investigate.

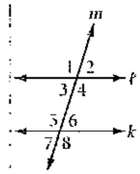


Investigation 1
Which Angles Are Congruent?

You will need

- lined paper
- a straightedge
- patty paper
- a protractor (*optional*)

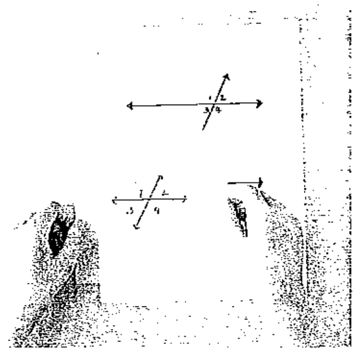
Using the lines on your paper as a guide, draw a pair of parallel lines. Or use both edges of your ruler or straightedge to create parallel lines. Label them k and ℓ. Now draw a transversal that intersects the parallel lines. Label the transversal m, and label the angles with numbers, as shown at right.



Step 1 Place a piece of patty paper over the set of angles 1, 2, 3, and 4. Copy the two intersecting lines m and ℓ and the four angles onto the patty paper.

Step 2 Slide the patty paper down to the intersection of lines m and k, and compare angles 1 through 4 with each of the corresponding angles 5 through 8. What is the relationship between corresponding angles? Alternate interior angles? Alternate exterior angles?

congruent or equal in measure



Guiding Investigation 1

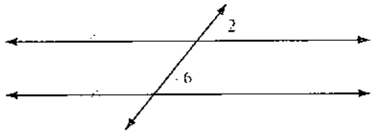
Steps 1, 2 You may work through Steps 1 and 2 as a follow-along activity. Students can use protractors to compare the angles if patty paper isn't available. Besides conjectures mentioning congruence, student conjectures might refer to angles as "shifts" of each other or even "flips" or "shifts and flips." Encourage a variety of ideas. (In fact, careful mathematical definitions of corresponding angles or alternate interior or exterior angles could be stated in terms of translations and reflections of the plane.) If students' conjectures refer to equal rather than congruent angles, save the critique for Sharing.

Compare your results with the results of others in your group and complete the three conjectures below.

Corresponding Angles Conjecture, or CA Conjecture

C-3a

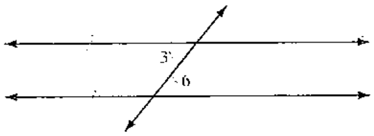
If two parallel lines are cut by a transversal, then corresponding angles are _?_. congruent



Alternate Interior Angles Conjecture, or AIA Conjecture

C-3b

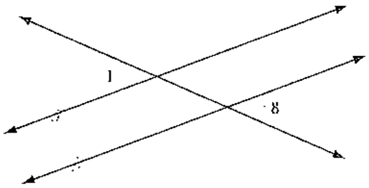
If two parallel lines are cut by a transversal, then alternate interior angles are _?_. congruent



Alternate Exterior Angles Conjecture, or AEA Conjecture

C-3c

If two parallel lines are cut by a transversal, then alternate exterior angles are _?_. congruent



The three conjectures you wrote can all be combined to create a Parallel Lines Conjecture, which is really three conjectures in one.

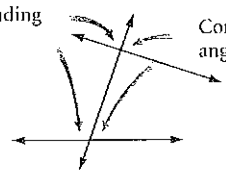
Parallel Lines Conjecture

C-3

If two parallel lines are cut by a transversal, then corresponding angles are _?_, alternate interior angles are _?_, and alternate exterior angles are _?_. congruent congruent congruent

Step 3 The conjecture is not true.

Step 3 What happens if the lines you start with are not parallel? Check whether your conjectures will work with nonparallel lines.



NCTM STANDARDS

CONTENT	PROCESS
Number	Problem Solving
Algebra	✓ Reasoning
✓ Geometry	✓ Communication
✓ Measurement	Connections
✓ Data/Probability	✓ Representation

LESSON OBJECTIVES

- Explore relationships of the angles formed by a transversal cutting parallel lines
- Learn new vocabulary
- Practice construction skills
- Develop inductive reasoning abilities, problem-solving skills, and cooperative behavior

Guiding Investigation 2

Step 2 If students have difficulty following the instructions, urge them to read carefully and follow one instruction at a time. The pair-share format of cooperative learning can be helpful.

Developing Proof
You might hand out the worksheet Alternate Interior Angles and ask students to close their books to ensure that they work on the proof without checking the solution. You can ask several groups to present their proofs during Sharing and then discuss the solution given in the book.

SHARING IDEAS

Choose students or groups to present a variety of conjectures. Lead a class discussion critiquing the conjectures until consensus is reached about which conjectures will get official names.

Be sure that any conjectures referring to "equal angles" are changed to contain the correct expression, "congruent angles" or "angles having equal measure."

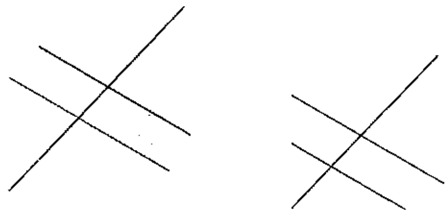
[Alert] Some students have difficulty with the three terms used in this lesson. Help students develop ways to remember the terms. For example: **[Ask]** "Why are the angles called *alternate interior angles*? *Alternate exterior angles*? *Corresponding angles*?" [It is because of their relationship to the transversal and the parallel lines.]

Some students may have drawn the transversal perpendicular to the parallel lines. For Investigation 1, they may have conjectured that other angles were congruent or right angles. Encourage critique of this idea. For Investigation 2, they may have conjectured that the lines were parallel. Elicit the idea that the conjecture is an if-then statement; unless the special case is included in the "if" part, the

What about the converse of each of your conjectures? Suppose you know that a pair of corresponding angles, or alternate interior angles, is congruent. Will the lines be parallel? Is it possible for the angles to be congruent but for the lines not to be parallel?

Investigation 2
Is the Converse True?

You will need
- a straightedge
- patty paper
- a protractor (*optional*)



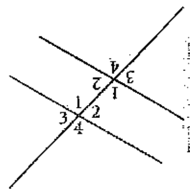
Step 1 yes

Step 1 Draw two intersecting lines on your paper. Copy these lines onto a piece of patty paper. Because you copied the angles, the two sets of angles are congruent.

Slide the top copy so that the transversal stays lined up.

Trace the lines and the angles from the bottom original onto the patty paper again. When you do this, you are constructing sets of congruent corresponding angles. Mark the congruent angles.

Are the two lines parallel? You can test to see if the distance between the two lines remains the same, which guarantees that they will never meet.



Step 2 alternate interior and alternate exterior angles; yes

Step 2 Repeat Step 1, but this time rotate your patty paper 180° so that the transversal lines up again. What kinds of congruent angles have you created? Trace the lines and angles and mark the congruent angles. Are the lines parallel? Check them.

Step 3 Compare your results with those of your group. If your results do not agree, discuss them until you have convinced each other. Complete the conjecture below and add it to your conjecture list.

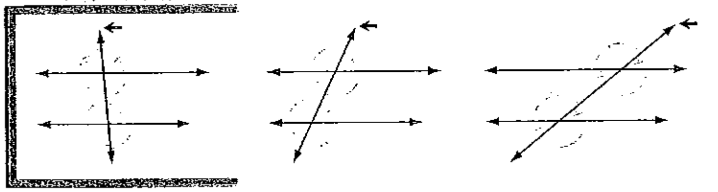
Converse of the Parallel Lines Conjecture C-4

If two lines are cut by a transversal to form pairs of congruent corresponding angles, congruent alternate interior angles, or congruent alternate exterior angles, then the lines are _?_. parallel

conjecture is not complete, even if the "then" part is true for the special case.

[Ask] "What kind of reasoning did you use to develop your conjectures?" [It was probably inductive.] If some students are at higher van Hiele levels, challenge them to prove their results deductively, assuming the Linear Pair and Vertical Angles Conjectures to be true. They won't be able to succeed unless they assume the truth of one of the three conjectures introduced in this lesson. If students are becoming too frustrated, direct them to the proof in the example. But remember that at this stage the book is modeling thinking at a higher van Hiele level. You should assess students based on their level of reasoning skills, not on skills they are not yet ready to use.

Close by discussing the importance of parallels in separating Euclidean geometry from non-Euclidean geometries.



keymath.com/DG

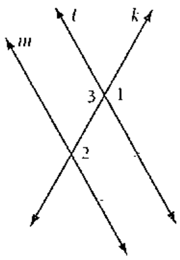
[▸ For an interactive version of both investigations, see the Dynamic Geometry Exploration Special Angles on Parallel Lines at www.keymath.com/DG. ◂]

Developing Proof You used inductive reasoning to discover all three parts of the Parallel Lines Conjecture. However, if you accept any one of them as true, you can use deductive reasoning to show that the others are true.

Read the example below. Before you read the solution, write a deductive argument with your group. Remember the reasoning strategy of applying previous conjectures and definitions. Then compare your solution to the one presented.

EXAMPLE

Write a deductive argument explaining why the Alternate Interior Angles Conjecture is true. Assume that the Vertical Angles Conjecture and Corresponding Angles Conjecture are both true.



▸ **Solution**

Deductive Argument

In the diagram, lines ℓ and m are parallel and intersected by transversal k. If the Corresponding Angles Conjecture is true, the corresponding angles are congruent.

$\angle 1 \cong \angle 2$

If the Vertical Angles Conjecture is true, the vertical angles are congruent.

$\angle 1 \cong \angle 3$

Because both $\angle 2$ and $\angle 3$ are congruent to $\angle 1$, they're congruent to each other.

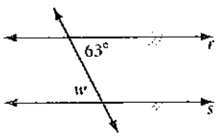
$\angle 2 \cong \angle 3$

Alternate interior angles 2 and 3 are congruent. Therefore, if the corresponding angles are congruent, then the alternate interior angles are congruent.

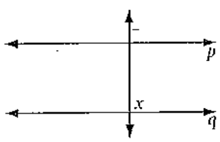
It helps to visualize each statement and to mark all congruences you know on your paper.

EXERCISES

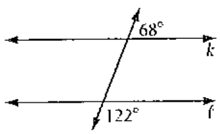
1. $w =$? 63°



2. $x =$? 90°



3. Is line k parallel to line ℓ? no



ASSIGNING HOMEWORK

Essential	1–8
Performance assessment	8
Portfolio	11
Group	10–13
Review	17–26
Algebra review	14–16, 23–26

MATERIALS

- Exercise 7 (T), *optional*
- Exercises 9 and 10 (T), *optional*

▸ *EXAMPLE*

This example shows how to prove one angle relationship from another deductively. The justification for "substitution" is that angles congruent to the same angle are congruent to each other. This is a good example of how more detailed theorems would have to be developed in order to use a two-column proof, often distracting from the concepts.

Assessing Progress

Assess students' abilities in collecting data, observing data, and making conjectures. Check their familiarity with parallel lines, angles, and vertical angles and their skill at using a protractor. Assess students' level of thinking. Those who easily understand a deductive proof when it is presented are at van Hiele level 2.

Closing the Lesson

Remind students of the important ideas: A **transversal** crossing two lines forms eight angles. Our conjectures say that if the lines are parallel, then the four pairs of **corresponding angles** are congruent, as are the two pairs of **alternate interior angles** and the two pairs of **alternate exterior angles.** Conversely, if one line crosses two others and if a pair of corresponding, alternate interior, or alternate exterior angles are congruent, we have conjectured that the two lines crossed are parallel. When we assume one of the first three conjectures, we can prove the other two deductively.

BUILDING UNDERSTANDING

The exercises apply the conjectures about angle relationships when a transversal cuts two parallel lines. Students often must extend lines to see the pairs of angles.

▶ Helping with the Exercises

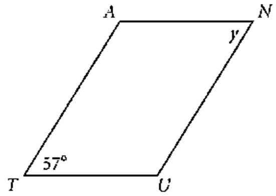
Exercise 4 Students may guess that the measure of angle y is the only measure given in the diagram, but to get them to justify that conjecture you may need to suggest that they extend some of the sides so that they can see transversals cutting parallel lines.

Exercise 5 From Exercise 4, students may conjecture that if two lines are parallel, then consecutive interior angles are supplementary. That conjecture alone does not provide a justification for the claim that quadrilateral *FISH* is a parallelogram; only the converse of that conjecture could be used. On the other hand, if the sides of quadrilateral *FISH* are extended, then a justification can be provided using the Converse of the Parallel Lines Conjecture.

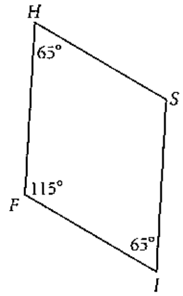
Exercise 6 Students can use alternate interior angles and linear pairs of angles.

Exercise 7 As students' conjecture lists grow, so does the number of conjectures needed to solve these angle-network exercises. Be sure students are aware that the angle measures need not be found in alphabetical order. If students get stuck, encourage them to find any angle measure they can, even if it's not labeled. These exercises, often called *angle-chase exercises,* are great activities for pair-share. Taking turns, one student finds an answer and explains why, then the other student finds an angle measure and explains why, continuing back and forth. After using pair-share with a few of these angle-chase exercises, you can introduce a variation. Taking turns, one student finds an answer and the second student explains why. Then the second student finds an angle measure and the first student explains why.

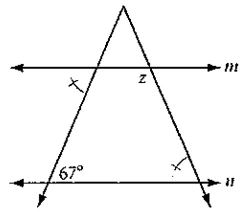
4. Quadrilateral *TUNA* is a parallelogram.
$y = \underline{?}$ ⓗ 57°



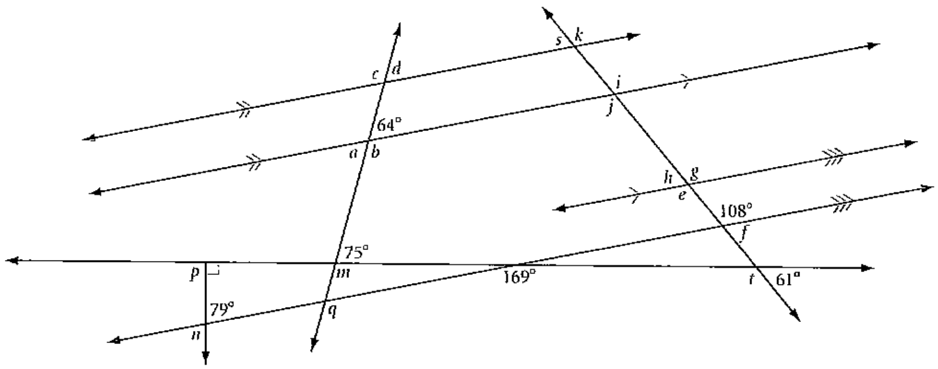
5. Is quadrilateral *FISH* a parallelogram? yes



6. $m \parallel n$
$z = \underline{?}$ ⓗ 113°

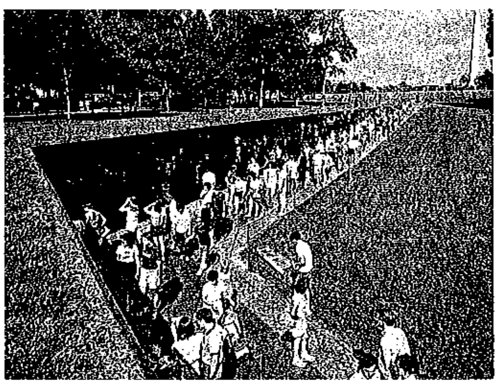


7. ***Developing Proof*** Trace the diagram below. Calculate each lettered angle measure. Explain how you determined measures n, p, and q. ⓗ

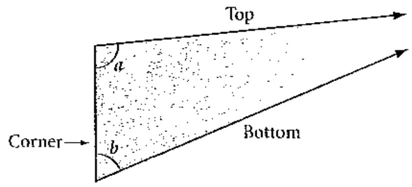


8. ***Developing Proof*** Write a deductive argument explaining why the Alternate Exterior Angles Conjecture is true. Assume that the Vertical Angles Conjecture and Corresponding Angles Conjecture are both true.

Cultural CONNECTION

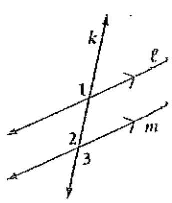


Sculptor Maya Lin designed the Vietnam Veterans Memorial Wall in Washington, D.C. Engraved in the granite wall are the names of United States armed forces service members who died in the Vietnam War or remain missing in action. Do the top and bottom of the wall meet in the distance, or are they parallel? How could you know from angle measures a and b in the diagram below? To learn more about the Memorial Wall and Lin's other projects, visit www.keymath.com/DG .

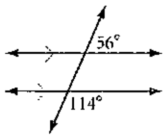


7. $a = d = 64°$, $b = c = 116°$, $e = g = i = j = k = 108°$, $f = h = s = 72°$, $m = 105°$, $n = 79°$, $p = 90°$, $q = 116°$, $t = 119°$; Possible explanation: Using the Vertical Angles Conjecture, $n = 79°$. Using the Linear Pair Conjecture, $p = 90°$. Using the Corresponding Angles Conjecture and $b = 116°$, $q = 116°$.

8. Possible answer: In the diagram, lines ℓ and m are parallel and intersected by transversal k. Using the Corresponding Angles Conjecture, $\angle 1 \cong \angle 2$. Using the Vertical Angles Conjecture, $\angle 2 \cong \angle 3$. Because $\angle 1$ and $\angle 3$ are both congruent to $\angle 2$, they must be congruent to each other. So $\angle 1 \cong \angle 3$. Therefore, if two parallel line are cut by a transversal, then alternate exterior angles are congruent.

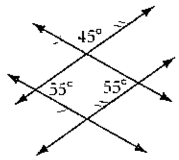


9. ***Developing Proof*** What's wrong with this picture?



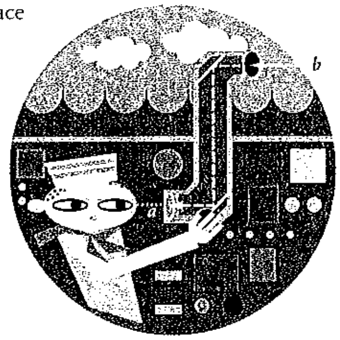
$56° + 114° = 170° \neq 180°$. Thus, the lines marked as parallel cannot really be parallel.

10. ***Developing Proof*** What's wrong with this picture?



Alternate interior angles measure 55°, but $55° + 45° \neq 180°$.

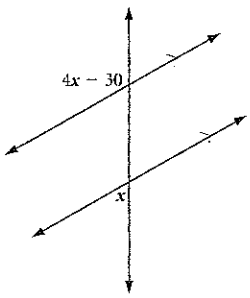
11. A periscope permits a sailor on a submarine to see above the surface of the ocean. This periscope is designed so that the line of sight a is parallel to the light ray b. The middle tube is perpendicular to the top and bottom tubes. What are the measures of the incoming and outgoing angles formed by the light rays and the mirrors in this periscope? Are the surfaces of the mirrors parallel? How do you know?



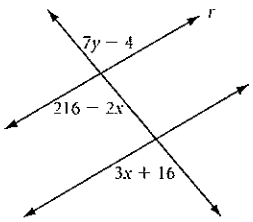
12. Draw a line on your paper and label it line AB. Place a point P about one to two inches from the line. Draw another line (a transversal) that passes through point P and line AB. Use your straightedge and protractor to draw line PQ that is parallel to line AB. Explain your method and why you know the lines are parallel.

13. Is the following statement true? "If yesterday was part of the weekend, then tomorrow is a school day." Write the converse of the statement. Is the converse true?

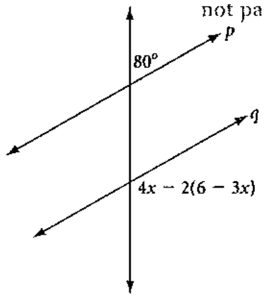
14. Find x. 42°



15. If $r \parallel s$, find y. 20°



16. If $x = 12°$, is $p \parallel q$? No, the lines are not parallel.



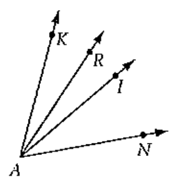
Review

.5 **17.** What type (or types) of triangle has one or more lines of symmetry? isosceles triangle

.6 **18.** What type (or types) of quadrilateral has only rotational symmetry? a parallelogram that is not also a rectangle or a rhombus

.1 **19.** If D is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$, what is the length of $\overline{AB}$ if $BD = 12$ cm? 18 cm

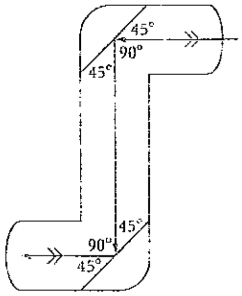
!.4 **20.** If $\overrightarrow{AI}$ is the angle bisector of $\angle KAN$ and $\overrightarrow{AR}$ is the angle bisector of $\angle KAI$, what is $m\angle RAN$ if $m\angle RAK = 13°$? 39°



!.3 **21.** If everyone in the town of Skunk's Crossing (population 84) has a telephone, how many different lines are needed to connect all the phones to each other? 3486

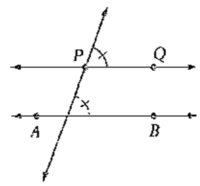
22. How many squares of all sizes are in a 4-by-4 grid of squares? (There are more than 16!) 30 squares (one 4-by-4, four 3-by-3, nine 2-by-2, and sixteen 1-by-1)

11. The incoming and outgoing angles measure 45°. Possible explanation: Yes, the alternate interior angles are congruent, and thus, by the Converse of the Parallel Lines Conjecture, the mirrors are parallel.



Exercise 12 Some students are too impatient to read instructions like this carefully. You might make copies of the problem for these students and suggest that they cross off each sentence as they do it. Then when their eyes return to the instructions, they can easily find where they were.

12. Explanations will vary. Sample explanation: "I used the protractor to make corresponding angles congruent when I drew line PQ."



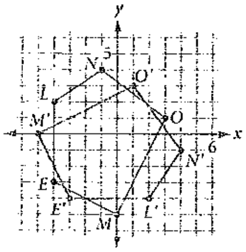
13. No, tomorrow could be a holiday. Converse: "If tomorrow is a school day, then yesterday was part of the weekend;" false.

Exercises 17, 18 These exercises review Chapter 0. If you skipped Chapter 0, you can use them to assess what students know about symmetry.

23. The triangle moved to the left 1 unit. Yes, congruent to original.

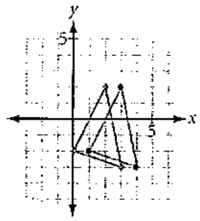
24. The quadrilateral was reflected across both axes or rotated 180° about the origin. Yes, congruent to original.

25. The pentagon was reflected across the line $x = y$. Yes, congruent to original.

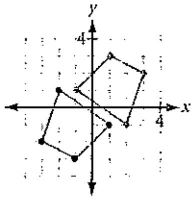


For Exercises 23–25, draw each polygon on graph paper. Relocate the vertices according to the rule. Connect the new points to form a new polygon. Describe what happened to the figure. Is the new polygon congruent to the original?

1.5 **23.** Rule: Subtract 1 from each x-coordinate.



1.6 **24.** Rule: Reverse the sign of each x- and y-coordinate.



1.6 **25.** Rule: Switch the x- and y-coordinates. Pentagon $LEMON$ with vertices:
$L(-4, 2)$
$E(-4, -3)$
$M(0, -5)$
$O(3, 1)$
$N(-1, 4)$

2.2 **26.** Assume the pattern of blue and yellow shaded T's continues. Copy and complete the table for blue and yellow squares and for the total number of squares.

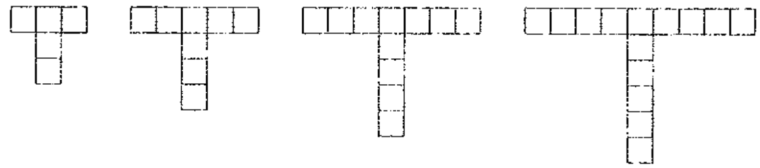


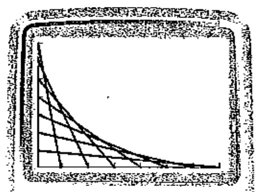
Figure number	1	2	3	4	5	6	...	n	...	35
Number of yellow squares	2	3	4	5	6	7	...	$n + 1$	...	36
Number of blue squares	3	5	7	9	11	13	...		...	71
Total number of squares	5	8	11	14	17	20	...		...	107

$3n + 2$ $2n + 1$

project

LINE DESIGNS

Can you use your graphing calculator to make the line design shown at right? You'll need to recall some algebra. Here are some hints.



1. The design consists of the graphs of seven lines.
2. The equation for one of the lines is $y = -\frac{1}{7}x + 1$.
3. The x- and y-ranges are set to minimums of 0 and maximums of 7.
4. There's a simple pattern in the slopes and y-intercepts of the lines.

Experiment with equations to create your own line design.

Your project should include

- A set of equations for the design shown here.
- Your own line design and a set of equations for it.

Project
You might use this project to assess authentically students' familiarity with equations of lines and with graphing calculators.

EXTENSIONS

A. Incorporate several sets of parallel lines within one drawing, and have students find missing angle measures.

B. Students could be challenged to think about "lines" on a cylinder. Lines are the shortest paths between points. [Some lines go "along" the cylinder, parallel to the center axis of the cylinder. Others are actually circles around the cylinder. Still others are helixes around the cylinder.] As on a plane, two of these "lines" are parallel if they don't intersect. Are the angle properties true for transversals crossing two parallel lines on a cylinder?

Supporting the project

In Chapter 0, some students created line designs by hand. Using this project as a starting point, they can create line designs on a graphing calculator.

OUTCOMES

- The equations for the lines are $y = -\frac{1}{7}x + 1$, $y = -\frac{2}{6}x + 2$, $y = -\frac{3}{5}x + 3$, $y = -\frac{4}{4}x + 4$, $y = -\frac{5}{3}x + 5$, $y = -\frac{6}{2}x + 6$, and $y = -7x + 7$.
- Student creates another line design and states equations that match the lines.
- Student creates several unique line designs.