Learning to Work with Matrices

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Length of lesson: 52 minutes
Grade/Level: 8th grade, Honors Algebra I
Source of the lesson: Holt Algebra I Florida Teacher's Edition textbook, 2004, Lesson 12.8 & Activity 6.3 from Mathematics: Modeling Our World, 1st. edition, by COMAP, and http://www.mrmyers.org/Brain_Busters/Matrix_Logic/race.html
Appropriateness for Middle School Students: Students will discover that a matrix is the technical term for tables they have used for years. They will manipulate different matrices to determine the values of added and subtracted matrices. Once the process is completed by hand, the students will attempt using a graphing calculator to solve the problem.
Concepts: Students will examine how multiple sets of data can be combined into one set by way of different operations.
Students will be able to:

- Calculate matrices using addition and subtraction by hand and by graphing calculator.
- Explain what makes two matrices equal.
- Identify the matrix dimensions and the individual matrix addresses.

Sunshine State Standards Addressed:

MA.912.D.8.1	Use matrices to organize and store data. Perform matrix operations (addition, subtraction, scalar multiplication, multiplication).
MA.912.D.8.2	Use matrix operations to solve problems.
MA.912.D.8.5	Use determinants of 2 x 2 and 3 x 3 matrices as well as higher order matrices with and without the use of technology.

Safety: There are no significant concerns.
Materials List and Advanced Preparations:

- Each student needs to have a pencil and paper
- 30 copies of the worksheets
- 30 graphing calculators
- Overhead calculator viewer
- Use of an overhead machine
- Transparencies
- Overhead projector
- Overhead marker

Pre-assessment	Time: 4 minutes
Pass out pre-assessment and explain that we don't necessarily expect them to know how to do it.	

Engagement		Time: 8 minutes
What the Teacher Will Do	Probing/Eliciting Questions	Student Responses Potential Misconceptions
Pass out the race engagement.	Here is a little brain teaser, try	

	to figure it out.	
Pass out WS1 and have a student fill in answers on T1 from his/her desk. Allow students to try on their own to fill out the page. Teacher circulates. Now go over it as a class. Explain the importance and the use of matrices.	So here we have a list of students and their scores on their first two tests. If we wanted to make our data more concise, how could we go about doing that, using what is given? How did you organize the data in the brain teaser? Okay, so Matrices have been along for a long time, we have found texts that date back to the ancient Chinese. But matrices are also used today, they are not only used in math, but also for designing computer game graphics, analyzing relationships, and even plotting complicated dance steps. We used a matrix in figuring out what order the students came in during the race. And here we just used matrices to analyze test scores easier.	Students may not know where/how to begin.
Exploration		**Time: 15 minutes**
What the Teacher Will Do	Probing/Eliciting Questions	Student Responses Potential Misconceptions
Students work individually on WS2. Teacher circulates and makes sure each student has the hang of it. Students have had time to reveal the message, call on a student to reveal the answer.	So now I am going to give you this worksheet and I want you to try to figure out the hidden message at the bottom. Okay, so who thinks they have deciphered the message?	Students may get the phrase from another student and complete the worksheet backwards. I got an A on the test!

Now explain the technical/vocabulary parts of matrices.	So in math we can arrange data in different ways, and using matrices is one way. A matrix is created by organizing our data in a special table of rows and columns which is enclosed by a set of brackets. Every matrix has a <u>dimension</u> and its dimension comes from the number of rows in the matrix, by the number of columns in the matrix. (row x column) So let's look at matrix A on the worksheet we just completed, What would its dimensions be? And how did you figure that out? Now, each spot inside the matrix is called an <u>entry</u> and each entry can be located by its <u>matrix address</u>. The address is made up of the matrix name (normally a designated letter) and the row and column of that entry. So who thinks they can tell me what this entry's address is (pointing to A_{11})	4 x 4 There are 4 rows and 4 columns. Receive both correct and incorrect answers. 6
Explanation		**Time: 5 minutes**
What the Teacher Will Do	Probing/Eliciting Questions	Student Responses Potential Misconceptions
Explain subtraction of matrices, the equality of matrices. Using (T3) to give examples of each idea.	So subtracting two matrices is done using the same method, only this time instead of adding the two corresponding entries we subtract. Pick a student to come solve the subtraction problem on the overhead. (see T3)	Students may want to make subtracting much more difficult than it is.

	Now, if I were to give you two matrices, and asked you if they were equal, what might you look at to decide if they are or are not equal? Choose multiple students for a variety of answers. Right, in order for two matrices to be equal BOTH the dimensions AND the corresponding entries must be equal. So then, are matrices *T* and *S* equal? (see T3) Why not? Okay, if I wanted to make matrix *M* equal to matrix *N*, what should the dimensions be for matrix *N*? Alright, then what would my entry be for N_{11}, ….ask for each entry.	Same values in the same spot, same dimensions. No 3 x 2
Continue to call on different students for the answers. Cover up matrix *M* with paper. Have a student create matrix *N* on the overhead.	Okay so who wants to come create matrix *N*?	

Elaboration		Time: 10 minutes
What the Teacher Will Do	Probing/Eliciting Questions	Student Responses Potential Misconceptions
Teacher will demonstrate a problem on the overhead calculator viewer to teach the students how to perform specific matrix operations using a graphing calculator. Instruct the students to just watch and not to write or use their own calculator at this time. Now put on overhead (T4) and have students do the two examples on the transparency	$A=\begin{bmatrix}-345 & 908\\ 245 & -321\end{bmatrix}$ $B=\begin{bmatrix}841 & -511\\ 721 & 612\end{bmatrix}$ $B-A=\begin{bmatrix}1186 & -1419\\ 476 & 933\end{bmatrix}$	Students may want to use the calculator at that time. Students' calculators may malfunction.

(using their graphing calculators). Circulate, help lost students.		
Evaluation/ Post-assessment		**Time: 10 minutes**
What the Teacher Will Do	Probing/Eliciting Questions	Student Responses Potential Misconceptions
Pass out the attached worksheet (WS.5). It will serve as their post-evaluation. Have the calculator instructions on the overhead.	Students are to attempt certain problems without a graphing calculator and others with the use of a graphing calculator.	

Pre Assessment: **Working with Matrices** Name_____ Date: _____

1. a.) Solve this matrix by hand. $\begin{pmatrix} 3 & 4 \\ 3 & 9 \\ 6 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 4 & 3 \\ 8 & 4 \end{pmatrix} = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$

   b.) What are the dimensions of this matrix? _____

2. a.) Solve this matrix by hand.

$A = \begin{pmatrix} 15 & 3 & 2 \\ 3 & -2 & 3 \\ 6 & -6 & -5 \end{pmatrix}$ $B = \begin{pmatrix} 19 & 3 & 6 \\ -3 & 9 & 1 \\ 21 & 3 & -5 \end{pmatrix}$ $A\text{-}B = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$

   b.) What is the matrix address for -5 in matrix *A*? _____

3. a.) Are matrices *S* and *T* equal? Circle: yes or no

$S = \begin{bmatrix} 7^2 & (1+9) \\ -2(10) & \frac{16}{2} \end{bmatrix}$ $T = \begin{bmatrix} 7(7) & \sqrt{100} \\ -(22-2) & 2^3 \end{bmatrix}$

   b.) Explain what is required for two matrices to be equal.

Engagement: **The Race!** Name_____ Date: _____

Six classmates raced one another on the way to the cafeteria. Use the matrix below to find out who came in first, second, third, fourth, fifth, and sixth.

	1st	2nd	3rd	4th	5th	6th
Sophie						
Andrew						
Cotter						
Rain						
Luke						
Peter						

1. Sophie crossed the finished line immediately after Luke.
2. Andrew finished right between Peter and Rain.
3. Cotter won the race.
4. Andrew came in right after the boy who finished second.
5. Luke saw four people finish before him.
6. Peter did not finish fourth.

Name_____ Date: _____

Discovering Matrices by Organizing Data

Use the student test scores to fill in the matrices working from left to right, top to bottom (like reading a book).

Student	Test 1 score	Test 2 score
Chris	86	94
Brittany	75	85
Michael	92	98
Brian	100	95
Michelle	83	92
Jackie	95	95

Student scores for Test 1

$$\begin{pmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{pmatrix}$$

+

Student scores for Test 2

$$\begin{pmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{pmatrix}$$

= the sum of the two test scores for each student

$$= \begin{pmatrix} \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_ \\ \_\_\_\_\_ & \_\_\_\_\_ & \_\_\_\_\_ \end{pmatrix}$$

How many points did each student get in both tests combined?

Chris: ____ Brittany: ____Michael: ____

Brian: ____ Michelle: ____Jackie: ____

Name_____ Date: _____

Solve the Secret Message Using Matrix Magic

Add matrices *A* and *B* together then use the key below to decipher the hidden message.

$$A = \begin{pmatrix} 6 & 3 & 7 & 12 \\ 5 & 2 & -3 & 16 \\ 7 & 2 & 6 & 3 \\ 10 & -2 & 12 & 11 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 4 & 8 & 8 \\ -4 & 12 & 4 & -1 \\ 7 & 18 & 5 & 4 \\ 10 & 7 & 7 & 9 \end{pmatrix}$$

$$A + B = \begin{pmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{pmatrix}$$

1=A	5=E	9=I	13=M	17=Q	21=U	25=Y
2=B	6=F	10=J	14=N	18=R	22=V	26=Z
3=C	7=G	11=K	15=O	19=S	23=W	
4=D	8=H	12=L	16=P	20=T	24=X	

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___!

Name_____ Date: _____

Matrices: Subtraction

Subtraction is done using the same method except this time we subtract the two entries instead of adding them.

$$A = \begin{pmatrix} 5 & 9 & 1 \\ 4 & 2 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 2 & 7 \\ 4 & 7 & 11 \end{pmatrix}$$

$$A - B = \begin{pmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{pmatrix}$$

Name_____ Date: _____

Matrices: Equality

Are matrices *S* and *T* equal?

$S = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 4 & 7 \end{pmatrix}$ $T = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 9 & 7 \end{pmatrix}$

1. Do they have the same dimensions?

   dimension of *S*_____ dimension of *T*_____

   Do they have the same dimensions? **Yes** **or** **No**

2. Are the corresponding entries equal? **Yes** **or** **No**

   If no, which entry is not equal? _____

❖ So are matrices *S* and *T* equal? **Yes** **or** **No**

I want to make matrix *N* equal to matrix *M*.

$M = \begin{pmatrix} 2 & 1 \\ 7 & 4 \\ -3 & 7 \end{pmatrix}$

a.) What should the dimensions be? _____

b.) What is my entry for N_{11} _____

N_{21} _____

N_{22} _____

N_{12} _____

N_{31} _____

N_{32} _____

$N =$ ( )

Calculator Directions

- ON
- HOME (picture of a house)
- CALCULATOR
- BOOK
- Go over to the 5th tab and hit ENTER
- Go to the 2nd row 5th picture over (M-BY-N MATRIX) press ENTER
- Put in the dimensions (use the TAB button to move), press ENTER.
- Input your given values then TAB out of the bracket.
- Indicate PLUS or MINUS function.
- Press BOOK and repeat above steps for the second matrix.

1. $A = \begin{bmatrix} 97 & -59 \\ 487 & 207 \end{bmatrix}$ $B = \begin{bmatrix} -2{,}439 & 4{,}567 \\ -311 & -3{,}462 \end{bmatrix}$

$A + B = \begin{bmatrix} & \\ & \end{bmatrix}$

2. $A = \begin{bmatrix} 194 & -91 \\ 7{,}485 & 1{,}328 \end{bmatrix}$ $B = \begin{bmatrix} -254 & 912 \\ 399 & -5{,}378 \end{bmatrix}$

$A - B = \begin{bmatrix} & \\ & \end{bmatrix}$

Post Assessment: **Working with Matrices** Name_____ Date: _____

1. a.) Solve this matrix by hand.

$$\begin{pmatrix} 2 & 1 \\ 4 & 6 \\ 3 & 9 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 3 & 1 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{pmatrix}$$

b.) What are the dimensions of this matrix? _____

2. a.) Solve this matrix by hand.

$$A = \begin{pmatrix} 12 & 2 & 7 \\ 9 & 4 & 4 \\ 7 & -5 & -6 \end{pmatrix} \quad B = \begin{pmatrix} 13 & 4 & 2 \\ 3 & -4 & 4 \\ 11 & 1 & -2 \end{pmatrix} \quad A - B = \begin{pmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{pmatrix}$$

b.) What is the matrix address for -5 in matrix *A*? _____

3. Solve using a graphing calculator.

$$\begin{pmatrix} -3{,}207 & 1{,}900 \\ 1{,}623 & 8{,}270 \end{pmatrix} + \begin{pmatrix} 3{,}294 & -210 \\ -273 & -3{,}908 \end{pmatrix} = \begin{pmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{pmatrix}$$

4. a.) Are matrices *S* and *T* equal?

$$S = \begin{bmatrix} 2^2 & (12-3) \\ -3(9) & \frac{24}{3} \end{bmatrix} \quad T = \begin{bmatrix} 2(2) & \sqrt{81} \\ -(30-3) & 2^3 \end{bmatrix}$$

Circle: yes or no

b.) Explain what is required for two matrices to be equal.