Math Objectives

- Students will interpret information represented in tables, graphs, and symbols.
- Students will be able to find the slope from a table, from a graph, and in an equation.
- Students will interpret the meaning of slope and intercepts in the context of a real world situation.
- Students will recognize that a real world situation often has conditions that naturally restrict the domain.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).
- Students will model with mathematics (CCSS Mathematical Practice).

Vocabulary

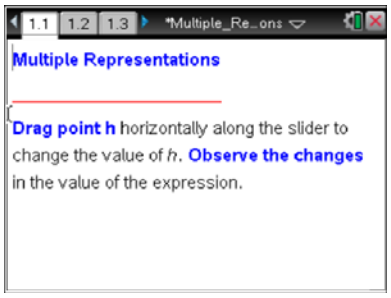
- function
- functional notation
- ordered pairs
- rate of change
- slope
- *y*-intercept

About the Lesson

- This lesson involves the concept of interpreting slope as a rate of change in the context of a real world problem situation.
- As a result, students will:
  - Collect ordered pairs and interpret the connections across various representations of the data using a symbolic rule relating income to the number of hours worked.
  - Investigate the connections between symbols, ordered pairs, tables, and graphs.
  - Recognize the vertical intercept and slope in each of these representations. The lesson assumes knowledge of function notation

TI-Nspire™ Navigator™ System

- Use Quick Poll to check student understanding.
- Use Screen Capture to examine patterns that emerge.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point
- Manually capture data

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Multiple_Representations_Student.pdf

Multiple_Representations_Student.doc

TI-Nspire document

Multiple_Representations.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

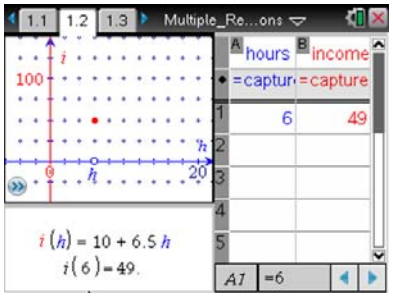
- Use Live Presenter to engage and focus students.
- Use Teacher Edition computer software to review student documents.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand (☜) getting ready to grab the point. Also, be sure that the word *point* appears. Then press ctrl ☜ to grab the point and close the hand (✊). When finished moving the point, press esc to release the point.

Teacher Tip: This lesson assumes that students are familiar with functional notation. It might be important to do a review to make sure they remember how it works.

Move to page 1.2.



Problem Situation: Tricia is a 17-year old student who receives an income from a weekly allowance and also works a part-time job paid at an hourly rate. She always receives her allowance, but on her job she can work any whole number of hours from 0 to 20 hours a week. The TI-Nspire document shows three representations of Tricia's possible income for one week depending on the number of hours she works.

1. Move *h* to zero, and press ctrl . once. What do you observe about the three representations?

   Answer: Function shows an output of 10 for the input 0. Table contains the ordered pair (0, 10). Graph contains the point (0, 10).

Teacher Tip: Students should understand that the *y*-intercept can be identified as the ordered pair (0, a) and can be found as the result of a symbolic computation, as an ordered pair by itself or in a table, or as the point where a graph intersects the vertical axis. This would also be a good point to talk about: the meaning of the *y*-intercept in the context of this

problem. If Tricia works 0 hours, she will still have an income of $10 because she gets her allowance whether she works any hours or not.
CCSS Mathematical Practice: Students will model with Mathematics. Mathematically proficient students should make the connection that the graph, equation, and ordered pairs are all mathematical models for Tricia's income from her weekly allowance and a part-time job. They should understand that manipulating the models represents or predicts changes that could occur in a real world situation.

2. Move *h* to eight, and press ctrl . once. Move *h* to another value, and press ctrl . once. Repeat. What do you observe about the three representations?

Answer: Function shows each individual input/output pair. Table shows all collected pairs. Graph plots all points.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

3. What do *h* and *i*(*h*) represent? Where do *h* and *i*(*h*) appear in each representation?

Answer: The *h* represents the number of hours Tricia works (independent variable) and appears as the input in the function, the first column in the spreadsheet, and the first coordinate of each point in the graph. *i(h)* represents Tricia's weekly income (the dependent variable) and appears as the output of the function, the second column in the table, and the second coordinate of each point in the graph.

Teacher Tip: This is a good opportunity to discuss the difference between a discrete and a continuous function. Although situations like this are typically modeled with a linear function, the relationship is not really continuous. The linear model is helpful for making calculations, but students should understand that the actual set of possibilities for Tricia's pairs of hours versus income is a collection of discrete points that all fall along a linear path.

4. Using your answer from question #3, how much would Tricia earn if she works eight hours? Where does this appear in each of the three representations?

Answer: $62 for eight hours. The table contains the ordered pair (8, 62). There is a point on the graph at (8, 62).

Teacher Tip: It is important for the students to understand that the reason for this is the symbolic relationship of the function rule. When you substitute eight (8) into the function, the result is 62. This ordered pair becomes a part of the data set in the table and a point on the graph.

CCSS Mathematical Practice: Students will make sense of problems and persevere in solving them.
Mathematically proficient students explain correspondences between what happens when Tricia works for eight hours in the equation, verbally, in the tables, and on the graph.

5. a. What is Tricia's allowance, and how can you identify this in each of the three representations?

Answer: Tricia's allowance is $10. The 10 is the constant value added in the function, the vertical intercept of the graph, and the second coordinate in the order pair (0, 10) in the table.

b. What is Tricia's hourly rate, and how can you identify this in each of the three representations?

Answer: Tricia's hourly wage is $6.50 per hour. The 6.50 is the coefficient of *h* in the function, the incremental increase of the output as h is increased by 1 hour, the slope of the graph, the rate of the change in the table.

Teacher Tip: It is important for students to understand that since the independent variable is measured in hours and the dependent variable is measured in dollars, the units for slope will be dollars per hour.

c. Why do the points in the graph fall on a straight line?

Answer: The relationship is linear defined by the starting point (0,10), her allowance, and the rate of change 6.5. Thus all of the points will fall on a straight line.

6. Can Tricia's income be $75 in one week? Explain.

Answer: Yes. She would have to work 10 hours to earn $75.

Teacher Tip: It is understood that many students may be content to just move the slider until the income reaches $75. You could probe deeper by asking about less "convenient" incomes, such as $100 (14 hours) or $50 (7 hours). Guide the discussion towards solving equations (75=10+6.6h,

100=10+6.5h, etc.). Students should also recognize that you can not simply round the number of hours to the nearest whole number. For example, when solving for \$50, h≈ 6.15 hours. However, Tricia must work 7 hours if she wants to earn \$50 because of the whole number requirement in the problem. Hence, you must always round up to find the number of hours required for a particular income.

7. Can Tricia earn \$400 in one week? Explain.

Answer: No. The most Tricia can make with the 20-hour domain restriction is \$140.

8. Why can you only move from 0 to 20 on the horizontal axis? What does this condition mean for each of the three representations?

Sample Answers: 0 is the minimum because Tricia can't work negative hours. 20 is the maximum because, in many states, students under the age of 18 cannot work more than 20 hours a week. This means that no ordered pair in the table or any point on the graph will contain an *h* value less than 0 or greater than 20, and all incomes will be between \$10 and \$140, inclusive.

Teacher Tip: Answers will vary, but the main issue addressed in the question is the idea that real world problems often have restrictions that require a different domain than the pure mathematical model used to represent them.

9. Is it possible for Tricia to have an income of \$0? Why or why not?

Answer: No. She will always have her allowance, so even if she works 0 hours, she will still earn \$10. This is reflected in the graph because the vertical intercept is (0, 10).

Teacher Tip: Depending on students' backgrounds, you might want to formally use the term *y*-intercept in describing the vertical intercept. Ask students if there is an *x* or horizontal-intercept. This provides an opportunity to discuss that while such a point exists on the linear function used to model the situation, it would indicate that she had worked a negative number of hours, which is not possible unless she had received an advance on her salary and so was working off the debt.

10. Suppose Tricia's parents increase her weekly allowance by \$5. Describe how this increase in allowance would affect each of the representations.

Answer: The constant added in the function would change from 10 to 15. The vertical intercept of the graph and corresponding ordered pair in the table would change to (0, 15).

11. Suppose Tricia gets a one dollar per hour raise. Describe how this increase in her hourly rate would affect each of the representations.

    Answer: The coefficient of *h* in the function would now be 7.50 showing the increase to $7.50 wage. 7.50 will now be the incremental increase of the output as h is increased by 1 hour. The slope of the graph, and the rate of change in the table will now be 7.50.

12. What do you think are the strengths and weaknesses of each type of representation?

    Sample Answer: Advantages: The function rule is a good way to describe the whole situation at once; function(input)= output and is very easy to calculate. The graph makes it possible to see that the points represent a linear relationship and the table allows you to reason from the exact numbers in the relation.
    Disadvantages: The function notation can be confusing; the graph does not show the exact ordered pairs; the table might not show all of the values and they do not come in any order unless you choose consecutive values for the input. And if the relationship is continuous, the table will not ever show all of the values. The function rule by itself suggests the function is continuous.

Teacher Tip: You can extend this lesson by having students actually edit the original function rule to match the conditions given in problems 10 and 11. To edit the function rule, double-click on the function rule, move to page 1.3, and type Define allowance= and enter the new value for the allowance. Press enter when complete. You can change her pay by typing define payrate= and entering the new pay rate. Students can collect more data points and investigate the representations further by comparing and contrasting the two data sets.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion of this activity, the teacher should ensure that students are able to:

- Identify the vertical intercept on a graph, in a table, or as an ordered pair.
- Recognize the slope of a line using a table or a graph.

- Recognize that when functions are evaluated they have many possible outcomes and can be represented using symbols, ordered pairs, tables, and graphs.

TI-Nspire Navigator™ Opportunities

- **Note 1 Screen Capture** As students are completing question 2, use the Screen Capture feature and check to make sure students are doing the activity properly and also you can show the class all the different points that can be selected.
- **Note 2 Quick Poll** Use the Open Response feature of Quick Poll and verbally ask students to answer the following questions:
  1. What does the y-intercept of this graph represent in the "real-world" problem?

     (possible answers: 10 dollars (include units here), her allowance, what she earns even if she doesn't work any hours)
  2. What does the slope of this graph represent in the "real-world" problem?

     (possible answers: six dollars and 50 cents per hour (include all units here), Tricia's hourly rate)
- **Note 3 Quick Poll** Have students answer the following questions and then submit them using the Open Response feature. Students will need to do some paper and pencil calculations. If you think that the students will need to use a calculator, have the students open the Scratchpad Calculator and do those calculations before sending the Quick Poll.
  1. If Tricia works 20 hours, how much will she earn? (answer: 20(6.50) + 10 = $140)
  2. If Tricia earns $49, how many hours did she work? (answer: 6.5 h + 10 = 49;

     h = 6 hours. Note: encourage the use of proper units in the answers)